Learner Pack

Level 4: Mathematics
Unit 1: Number

Activity Decorating Code N1

This activity links to award learning outcome 1.4.

Introduction

Scientific calculators are very useful at performing difficult calculations. Scientific calculators can compute square roots, fractions, indices and many more operations.

Materials you will need

Scientific Calculator

Learning Outcomes

1. Perform addition, subtraction, multiplication and division on a calculator.
2. Use the calculator to compute fractions, percentages and square roots.
3. Use scientific notation and symbols such as π.
4. Recognise and apply the order of operations.

Key Learning Points

1. Calculator

What do you need to know before you start?

Maths

π (Pi) is a mathematical symbol. This symbol is a letter from the Greek alphabet. In mathematics it represents the fraction $\frac{22}{7}$ which is the decimal 3.14. There is a π button on the scientific calculator. The fraction $\frac{22}{7}$ is a special number that is useful in Geometry.

Tip:

Draw a few different sized circles. Measure the length of the circumference of each circle, using a piece of string. (See the Level 3 Pack, Application of Number, which explains how to do this). Then measure the diameter of each circle. Now divide the circumference by the diameter.

No matter how big or small your circle is, the circumference divided by the diameter is always π.

Remember

When doing more than one operation you must follow the correct order of operations – **BIDMAS.**
For example:

27 + 14 – 2 x 5 = 27 + 14 – (10)
= 27 + 14 – 10
= 41 -10
= **31**

(See the Level 3 pack for more on BIDMAS)

Task 1: Percentages and Fractions

Example

Two shops, H & N and Lara, both stock the latest pair of Jimmy Choo high heel shoes. The retail price of these shoes is €127.

H & N is offering a 13% discount. Lara is offering $\frac{1}{7}^{th}$ off the price.

How much will the shoes cost at H & N?

How much will the shoes cost at Lara?

Which shop would you buy the shoes in?

Solution

How much will the shoes cost at H & N?

H & N is offering 13% off the shoes which cost €127.

13% = $\frac{13}{100}$

$\frac{13}{100}$ of 127:

Use your calculator to work it out.

If your calculator is a Sharp calculator press the following:

13 fraction 100 right arrow x 127=

If your answer is in the form of a fraction then press change and you will get a decimal.

If your calculator is a Casio calculator press the following:

Fraction 13 down arrow 100 right arrow x 127 =

If your answer is in the form of a fraction then press the S⇔D button and you will get a decimal.

This will give you an answer of **16.51**. This means that H & N are offering €16.51 off the price of the shoes. Therefore at H & N the shoes will cost €110.49.

127 – 16.51 = 110.49

How much will the shoes cost at Lara?

Lara is offering $\frac{1}{7}^{th}$ off the price of the shoes which cost €127.

$\frac{1}{7}$ of 127:

Use your calculator to work it out.

If your calculator is a Sharp calculator press:
1 fraction 7 right arrow x 127 =

If your calculator is a Casio calculator press:
Fraction 1 down arrow 7 right arrow x 127=

This will give you an answer of 18.14. This means that Lara are offering €18.14 off the price of the shoes. So at Lara the shoes will cost €108.86.
127 – 18.14 = 108.86

Which shop would you buy the shoes in?

What did you decide? In this case, the same shoes cost €110.49 in H & N and €108.86 in Lara, so Lara is a little cheaper. However, price is just one thing we think of when deciding which shop to buy in.

Task 2: Multiplication & Division

Example

You are carpeting you sitting room (which is a rectangular room). The length of your room is 5.45m and the width is 3.78m.

Use your calculator to try and work out the following:

(i). How much carpet will you need to buy?

(ii). The carpet costs €17.93 per m^2. How much will the carpet cost?

Solution

(i) The area of the floor in your sitting room is (length) x (width)
Area = 5.54 x 3.78

In your calculator press **5.54 x 3.78 =**

This will give you an answer of 20.94m^2.

Therefore you need **20.94m^2** of carpet.

(ii) You need 20.94m^2 of carpet and each m^2 costs €17.93.

In your calculator press **20.94 x 17.93 =**

This will give you an answer of 375.4542. Therefore it will cost you €375.45 to carpet your room.

Task 3: Square Roots

Example

You also wish to replace the tiles in your bathroom. Your bathroom is a **square** room and the area of its floor is a square of 5.76 m².

What is the length of the bathroom?

Solution

The area of the floor in the bathroom is (length) x (width). As it is a **square** room the length is the same as the width: so the area is (length)². We know that the area is 5.76m² .

Area = (length)².

5.76m² = (length)²

So, to find the length we need to find the square root of **both sides.**

$$\sqrt{5.76\text{m}^2} = \sqrt{(\text{Length})^2}$$

$$\sqrt{5.76m^2} = \text{Length}$$

Use your calculator to work it out:

In your calculator press $\sqrt{\ }$ then press **5.76 =**

This will give you an answer of 2.4. Therefore the length (or width) of your bathroom is **2.4m**.

Task 4: π

Example

If you wanted to put a circular pond in the garden you can buy plastic fittings to make the circular pond outline. They come in various diameter sizes. You would need to decide what area of ground you should dig for the pond.

If you want a pond which is 10m in diameter, what should the ground surface area of the hole you dig be?

Solution

(i) The area of a circle is πr^2. The radius of a circle is half its diameter. Therefore because the diameter is $10m^2$ the radius is 5m.

Area = πr^2

Area = $\pi(5)^2$

Using your calculator:

If you have a Sharp calculator then you use the button y^x to work out powers.

If you have a Casio calculator then you use the button $x^{\square}$ to work out powers.

In your calculator press **5 power button 2 =**

This will give you an answer of 25. Now you have:

Area = $\pi 25$

In your calculator press π **x 25 =**

This will give you an answer of 75.5398. Therefore the area of the circular pond with a radius of 5m is **$78.54m^2$.**

Now you try this.

A recipe for chocolate cake says you need a baking tray with an area of 12.57cm. You have two baking trays. One has radius of 2cm and the other has a radius of 3cm. Which baking tray should you use?

Sean is sharing a flat with two other guys. He pays one third of the gas bill and 38% of the electricity bill but does not pay the Cable TV bill. This month's gas bill is €76.50 and the electricity bill is €64.20. How much does Sean pay for the gas bill and for the electricity bill?

How much change should Sean's flatmate give him if he left 3 twenty euro notes towards his share of the bill?

Practise your skills

Practice Sheet N1

Activity **The Telethon** **Code N2**

This activity links to award learning outcome 1.1 and 1.4.

Introduction

The 'People in Need Telethon' is a fundraising event that is held each year to raise money for charities in Ireland. People from around the country think of unique ways to raise funds to help those most in need. RTE broadcast an event each year to mark the People in Need Telethon and people get a chance to phone or text in to say how much money they raised and how they did it.

Materials you will need for this activity

1. Calculator
2. Pen & Paper

Learning Outcomes

1. Multiply by three digit numbers.
2. Divide by three digit numbers.

Key Learning Points

1. Three Digit Numbers
2. Multiplication
3. Division

What do you need to know before you start?

Telethon

- A telethon is a series of events to raise money for charity, combined with a TV broadcast about the events and about the charity.

How can you find out more about this?

- Look up www.telethon.ie
- Ask friends/family members.

Maths

Multiplication

During Level 3 you would have learnt about the concept of multiplication and seen how 3 x 4 = 3 + 3 + 3 + 3 = 12.

Often we are required to multiply and divide by much larger quantities. It is important to remember that the idea of multiplication and division never changes and when asked to calculate 25 x 13 we are still trying to work out 25 added to itself 13 times. If we were to write this out and calculate it in this way it would take us an extremely long time and so we use what we know about multiplication and place value to help us work it out.

First we must organise the figures so the units are in one column and the tens are in another as so:

```
  25
x 13
```

We then multiply the units (5 x 3 = 15). We have 5 units (which we place in our units column and 1 ten which we carry over to our ten column:

```
  25
x 1(1)3
   5
```

Now multiply the units by the tens (3 x 2 = 6 and add our additional ten: 6 + 1 = 7)

```
  25
x 13
  75
```

Now we must multiply our tens. We put a zero in the units column as we are focusing just on the tens and then multiply our tens by our units (1 x 5 = 5):

```
  25
x 13
  75
  50
```

Finally we multiply the tens by the tens (1 x 2 = 2) and add the two answers we have to complete the sum:

```
  25
x 13
  75
 250
 325
```

We do the exact same when multiplying by larger say three digit numbers. For example:

```
  125
x 155
  625
 6250
12500
19375
```

Division

Also in Level 3 we studied the idea of division and saw that 14 ÷ 7 = 2 is the same as asking how many 7's are in 14.

As with multiplication, when dividing larger numbers the concept doesn't change. If asked to calculate 180 ÷ 15 we are looking for the number of 15's in 180. To do this we break it down into simpler division:

180 ÷ 15:

First we know that 1 is not big enough to be divided by 15 and so we start dividing 18 by 15.

There is one 15 in 18 with a remainder of 3 (18 – 15 = 3).

Now we bring over our 3 to our units and divide 15 into 30. We know there are 2 15's in 30 with no remainder.

Therefore 180 ÷ 15 = 12:

```
    12
15 |180
   -15
    30
  - 30
     0
```

Again as the numbers increase in size the concept remains the same.

1500 ÷ 125 = 12

```
     12
125 |1500
   -125
     250
  -  250
      0
```

Now try the tasks on the next few pages!

Task 1: Calculating your donation

Samantha decided to raise money for Telethon in 2009 and set out with the intention of raising €3520. Her idea was to hold a concert in the local school and charge everybody €32 euro to attend. She managed to convince Dustin the Turkey to host the event for free. On the first night 46 people attended the event. On the second night 52 people attended.

(i) How much money was made on the first night?
(ii) How much extra money was made on the second night?
(iii) Unfortunately Samantha did not reach the original target she set? How many tickets would she have to have sold in total to reach this target?

Solution

(i) On the first night 46 people attended.

If each ticket cost €32 then the total money raised was 46 x 32.

$$\begin{array}{r} 46 \\ \times\ 32 \\ \hline 92 \\ 1380 \\ \hline 1472 \end{array}$$

On the first night €1,472 was raised.

(ii) On the second night 52 people attended.

Again each ticket cost €32 so the total raised was 52 x 32.

$$\begin{array}{r} 52 \\ \times\ 32 \\ \hline 104 \\ 1560 \\ \hline 1664 \end{array}$$

Therefore on the second night €1,664 was raised.

The extra money raised on the second night:

1664-1472 =€192

(iii) The original target set by Samantha was €3520. In order to find out how many tickets needed to be sold to reach this target we must find how many €32 are in 3520.

$$
\begin{array}{r|l}
 & 110 \\
\hline
32 & 3520 \\
 & -\ 32 \\
 & \ \ 32 \\
 & -\ 32 \\
 & \ \ 00 \\
 & -\ \ 0 \\
 & \ \ \ 0
\end{array}
$$

Therefore in order to reach the target 110 tickets had to be sold.

Now you try this.

- Samantha decided to hold a concert again in 2010 to raise money for the relief work in Haiti. Due to the recession Samantha reduced her prices in 2010 to €27 and set a new target of €2565.

(i) How many tickets did Samantha have to sell this time to reach her target?

(ii) On the first night 37 people attended the concert while on the second night 89 people attended. How much money was raised in total?

Practise your skills

- Practice Sheet N2

Related Activity N2

Jenny works in a company which produces soft toys. Each month Jenny receives €2 for every soft toy she makes and €100 bonus if she makes over 279 soft toys per month.

In March how many toys must Jenny produce a day in order to get her €100 bonus?

Last November Jenny was out sick for 2 weeks and as a result only managed to make 152 toys in that month. What was her gross pay for that month?

In December Jenny must produce a greater number of toys and so she gets a €150 bonus if she produces over 341 toys. How many toys must she produce a day in December to get her bonus?

If Jenny produces 350 toys in December what is her gross pay?

Activity Taxing Goods Code N3

This activity links to award learning outcomes 1.1, 1.2 and 1.7.

Introduction

Value Added Tax (VAT) is a tax charged on the sale of goods. It is included in the price of most goods and services that we use every day. As of January 2010 the standard rate of VAT was 21%. However a reduced rate of VAT (13.5%) is charged on goods such as fuel, electricity and concert tickets. No VAT is applied to tea, coffee, milk, bread, books, medicine and children's clothes and shoes.

Materials you will need

Information about the different rates of VAT. This is available on http://www.citizensinformation.ie/en/money_and_tax/tax/duties_and_vat/value_added_tax.html)

- Calculator
- Pen & Paper

Learning Outcomes

1. Understand percentages.
2. Appreciate the everyday uses of percentages.
3. Convert percentages to fractions.
4. Calculate VAT payable.

Key Learning Points

1. VAT
2. Percentages

What do you need to know before you start?

Almost every day some newspaper article or advertisement refers to a percentage of some kind. For example a headline might read *"20% increase in knife crime"* or an advertisement for a clothes store may say "50% off selected lines this week only".
These percentages are simply another way of expressing fractions. So the ad that reads "50% off selected lines this week only" could also read "Selected lines ½ price for this week only", because 50% = ½.
The term **per cent** means per one hundred. Therefore 50% really means 50 in every hundred, or $^{50}/_{100}$ which breaks down to a half, ½.

In order to find a certain percentage of a number – for example, 50% of 20 - we must first convert the percentage to a fraction. In order to do this we write the percentage over a hundred and simplify if possible. For example:

$$26\% = \frac{26}{100} = \frac{13}{50}$$

$$10\frac{1}{2}\% = \frac{10.5}{100} = \frac{105}{1000} = \frac{21}{200}$$

Taxing Goods

VAT is a tax that is put on goods and services. You could find out about:

- different rates of VAT
- the rate of VAT which applies to different goods and services
- the equivalent of VAT in other regions such as the United States.

How can you find this out?

- Use the internet
- Use information available through Citizens Information
- Inquire with local shop owners or service providers
- Ask family members or friends.

Task 1: Rates of VAT

Example

There are **three different rates of VAT** in Ireland (2010). The highest rate of VAT is **21%** and applies to the majority of goods and services such as alcohol, audio-visual equipment, car parts and accessories, CDs, computers, consultancy services, cosmetics, detergents, diesel, fridges, furniture and furnishings, hardware, jewellery, lawnmowers and machinery. The next rate is **13.5%:** this applies to fuel (coal, heating oil, gas), electricity, restaurants, admission to cinema, music concerts and museums, veterinary fees, building and building services, agricultural contracting services, newspapers, short-term car hire, cleaning and maintenance services. Finally a rate **of 4.8%** is specifically for agriculture and applies to livestock (excluding chickens), greyhounds and the hire of horses.

Convert the three different rates of VAT to fractions in their simplest form.

Solution

Rate 1 = 21%

To convert this to a fraction we must place 21 over a hundred and simplify if possible

$\Rightarrow 21\% = \frac{21}{100}$ (This cannot be simplified any further)

Rate 2 = 13.5%

$\Rightarrow 13.5\% = \frac{13.5}{100} = \frac{135}{1000}$

135 and 1000 have a common factor of five and so we can say that

$13.5\% = \frac{27}{200}$

Rate 3 = 4.8%

$\Rightarrow 4.8\% = \frac{4.8}{100} = \frac{48}{1000}$

48 and 1000 have a common factor of eight and so we can say that

$4.8\% = \frac{6}{125}$

Now you try this.

In the United States the State Governments apply a General Sales Tax (GST) to products. This tax is quite similar to VAT (Value Added Tax).

However different States have different tax rates. The tax rates for a select number of States are detailed in the table below:

State	**GST Rate**
Alabama	4%
Idaho	6%
California	8.25%

Write these three tax rates as fractions in their simplest form.

Task 2: Pricing a good

Example

A retail electrical store wants to buy in some plasma screen HD televisions from the manufacturers. The manufacturers quote a price of €290 **excluding VAT**.

The VAT applied to televisions is 21%.

- How much tax is paid on each television?
- What is the new cost of the television?

Solution

In order to calculate how much VAT must be paid we must find 21% of 290.

21% = $\frac{21}{100}$ and so we must find $\frac{21}{100}$ of €290.

If you recall from studying fractions if we are looking to find a fraction <u>of</u> a number we must multiply this number by the fraction.

$\Rightarrow \frac{21}{100}$ of €290 = $\frac{21}{100}$ x €290

$= \frac{6090}{100}$

⇒ The amount of VAT to be paid on each Plasma screen HD television will be €60.90.

As a result the new price of the television will be 290 + 60.90 = €350.90

Now you try this.

- The manager of the electrical store purchases the 10 televisions from the manufacturer at a price of €350.90 per telly. However before he can sell them he must also add VAT of 21% to the good which the customers much pay. How much VAT will a customer who buys one of the new televisions have to pay?

- What is the new price of the television?
- The profit which the electrical store will make on each television is 5% of the price of the television including VAT (Your answer from the last question!). How much profit will they make on each television?

Practise your skills

- Practice Sheet N3

Related Activity N3

Shane was recently involved in a car accident and although no one was injured his car was damaged in the crash. Once it was fixed the mechanic posted the following invoice to Shane, but he never filled in the total amount or the VAT that would be charged on the parts and the labour. **Complete the invoice below and calculate the total cost of fixing Shane's car.**

Item	**Cost**
New bumper	€165
New windscreen	€226
New tyres	€180
New Wing Mirror	€129.90
Total Cost of Parts (Excluding VAT)	
VAT (@ 21%)	
Total Cost of Parts (Including VAT)	
Total Cost of Labour (Excluding VAT)	€323.20
VAT (@ 13.5%)	
Total Cost of Labour (Including VAT)	
Total Cost of Labour & Parts	

Activity He Shoots, He Scores!! Code N4

This activity links to award learning outcomes 1.1, 1.2 and 1.7

Introduction

The only way to win a game of soccer is to score more goals than the opposition. So it is important that lots of different players in any team are capable of scoring goals. Many teams suffer as they depend too much on their strikers to score the goals. However, some of the best teams have strikers, midfielders and even defenders who can score goals for them.

Materials you will need

- Calculator
- Pen & Paper
- Statistics from the English 2009/2010 Premier League season (Available on http://www.premierleague.com/page/Statistics)

Learning Outcomes

1. Understand how to convert fractions to decimals.
2. Write word problems in mathematical form and solve them.

Key Learning Points

1. Word Problems
2. Conversion

What do you need to know before you start?

You will often hear people say things like “I got 10 out of 15 right” or “I collected 20 of the 25 souvenir magazines”. When people say this they are simply putting fractions into words. For example if you got ten out of fifteen in an exam you would write this as $\frac{10}{15}$ or $\frac{2}{3}$. That is, you are saying you got $\frac{2}{3}$ of the exam correct. In the same way, if you said you collected 20 of the 25 souvenir magazines you could write this as $\frac{20}{25}$ or $\frac{4}{5}$, and say that you collected $\frac{4}{5}$ of the magazines.

In order to then **change these fractions to a percentage** we must remember what the term percentage means. The term **per cent** means ‘per hundred’. For example 40% really means forty out of a hundred or $\frac{40}{100}$.

To change $\frac{40}{100}$ back to a percentage we would multiply it by 100.

Therefore **to convert any fraction to a percentage we multiply the fraction by $\frac{100}{1}$ %** and simplify. For example:

$\frac{1}{2} = \frac{1}{2} \times \frac{100}{1}\% = \frac{100}{2}\% = 50\%$

$\frac{2}{5} = \frac{2}{5} \times \frac{100}{1}\% = \frac{200}{5} = 40\%$

For this activity it would also be useful to know:

- top scorers in the English Premiership in the 2009/2010 season,
- the midfielder with the highest number of goals in the English Premiership in the 2009/2010 season.

How can you find this out?

- Use the internet
- Ask family members/friends

Task 1: Chelsea's Firepower

Example

Chelsea won the English Premier League in the 2009/2010 season and scored an impressive 103 goals. 4 of their players were among the top 20 goal scorers in the Premier League that season and only two of these were strikers. Didier Drogba, one of their two main strikers, scored 29 of the 103 goals. Their second striker, Anelka, scored 11 of the 103 goals. Two of their midfielders also scored more than 10 goals each. Frank Lampard scored 22 goals and Florent Malouda scored 12 goals.

Try working these questions out before you look at the solutions.

1. What fraction of Chelsea's total amount of goals did Frank Lampard score?

2. What percentage (to the nearest whole number) of goals did Didier Drogba score?

3. Did Drogba and Lampard, between them, score more than half of Chelsea's goals in 2009/2010?

Chelsea's Firepower

Solutions

1. We were told that Lampard scored 22 goals.

In total Chelsea scored 103 league goals.

So, Lampard scored $\frac{22}{103}$ of Chelsea's goals in 2009/2010.

2. Drogba scored 29 of the 103 goals.

So, Drogba scored $\frac{29}{103}$ of Chelsea's goals in 2009/2010

To convert this to a percentage we must multiply by $\frac{100}{1}$ %

$\frac{29}{103}$ x $\frac{100}{1}$ % = $\frac{2900}{103}$ % = 28.15% = 28%.

3. If Lampard and Drogba scored more than half of Chelsea's goals then they would have to have scored over 50% of the goals.

Remember ½ = 50%

We know Drogba scored 28% so we must calculate what percentage Lampard scored i.e. convert $\frac{22}{103}$ to a percentage.

$\frac{22}{103}$ x $\frac{100}{1}$ % = $\frac{2200}{103}$ % = 21.35% = 21%

⇒ In total Lampard and Drogba scored 28 + 21 = 49% of Chelsea's goals and so we can say that they **did not** score more than half of Chelsea's goals in 2009/2010

Now you try this.

What fraction of Chelsea's goals did Florent Malouda score in 2009/2010?

What percentage of goals did Anelka score for Chelsea in the league in 2009/2010?

Which combination scored the greater percentage of goals?

a. Drogba & Anelka or Lampard & Malouda

b. Drogba & Malouda or Anelka & Lampard

Practise your skills

- Practice Sheet N4

- Class Activity 1:
  The top four scores in the Premier League in 2009/2010 were
  1. Didier Drogba (Chelsea: 29 goals),
  2. Wayne Rooney (Man United: 26 goals),
  3. Darren Bent (Sunderland: 23 goals),
  4. Carlos Tevez (Man City: 22 goals).
  Find out how many goals their teams scored in the season. Calculate what percentage of the teams' total goals was scored by these players.
- Class Activity 2:

Which team won the most recent Premier League in England? Choose a team each and find out how many goals it scored in total during that league. Calculate **what percentage of the team's total goals was scored by their top scorer.**

Which player scored the largest percentage of their team's goals?

You can find the statistics you need on
http://www.premierleague.com/page/Statistics/0,,12306,00.html

Activity National Television Awards Code N5

This activity links to award learning outcomes 1.1 and 1.2.

Introduction

The National Television Awards ceremony is held once a year. It rewards actors who appear in television soaps or documentaries. Some of the awards such as best soap actor, best soap villain and overall best soap are decided upon by a public vote: members of the public ring in to vote for their favourite. In 2010 Coronation Street won the award for Best Soap and the X Factor was crowned Most Popular Talent Show.

Materials you will need

- Calculator
- Pen & Paper

Learning Outcomes

1. Understand the relationship between decimals, percentages and fractions.
2. Convert decimals to fractions.
3. Convert decimals to percentages.

Key Learning Points

- Decimals, percentages and fractions
- Conversion

What do you need to know before you start?

Maths

We know from Level 3 mathematics that decimals are simply another way of writing fractions. We saw that fractions could also be written as percentages. So we know that percentages, fractions and decimals all represent the same idea.

A mixture of decimals, percentages and fractions often appear in the same piece of information or in the same maths problem. So it is important for us to be able to convert from one form to the other.

Converting decimals to fractions

When looking at decimals we saw how each digit has a place value: for example, tenths, hundredths or thousandths. If there is one digit to the right hand side of the decimal point we place it over ten and simplify. Remember that digits immediately to the right of the decimal point are tenths. For example:

$0.6 = \frac{6}{10} = \frac{3}{5}$.

$3.5 = \frac{35}{10} = \frac{7}{2} = 3\frac{1}{2}$

Similarly if there are two digits to the right hand side of the decimal point we place the number over a hundred and simplify. For example:

$0.45 = \frac{45}{100} = \frac{9}{20}$

$5.26 = \frac{526}{100} = \frac{263}{50} = 5\frac{13}{50}$

Finally if there are three digits to the right hand side of the decimal point we place the number over a thousand and simplify. For example:

$0.765 = \frac{765}{1000} = \frac{153}{200}$

$2.375 = \frac{2375}{1000} = \frac{19}{8} = 2\frac{3}{8}$

To convert a decimal to a percentage we must first convert it to a fraction and then convert this fraction to a percentage. We do this b multiplying our fraction by $\frac{100}{1}$%. For example:

$0.25 = \frac{25}{100} = \frac{1}{4} = \frac{1}{4} \times \frac{100}{1}\% = \frac{100}{4}\% = 25\%$

$0.125 = \frac{125}{1000} = \frac{1}{8} = \frac{1}{8} \times \frac{100}{1}\% = \frac{100}{8}\% = 12.5\%$

National Television Awards

- Winners of different awards 2010.
- How the winner of each award was selected i.e. was it public vote or did a panel of judges decide.

How can you find this out?

- Use the internet
- Find clips from the Awards Ceremony (They were held on the 20th January 2010)
- Ask family members/friends.

Task 1: Coronation Street

Example

Coronation Street won the award for the best soap at the British National Television Awards in 2010. This decision was based on votes submitted by members of the public.

When the results were analysed it was found that Coronation Street only just beat their main rivals, Eastenders. Coronation Street got 0.32 of the public vote. $\frac{3}{10}$ of the votes cast were for Eastenders, Hollyoaks got 13% of the vote. 0.18 of the vote was for Doctors, while 0.07 of the votes were for Holby City.

Calculate what fraction of people voted for Holby City and Doctors.
Simplify your fractions if possible.

Solution

We know that 0.18 of the vote was for Doctors.

$0.18 = \frac{18}{100}$

18 and 100 have a common factor of 2

$0.18 = \frac{9}{50}$

We also know that 0.07 of the vote was for Holby City

$0.07 = \frac{7}{100}$

This cannot be simplified

So we know that $\frac{9}{50}$ of the votes cast were for Doctors while $\frac{7}{100}$ of the votes were for Holby City.

Now you try this.

What fraction of the vote cast was Coronation Street?

What percentage of voters voted for Eastenders?

Task 2: Most Popular Talent Show

Example

In the British National Television Awards in 2010, 350,000 votes were cast in the Most Popular Talent Show category. In total four different talent shows were voted for. These were Britain's Got Talent, X Factor, Dancing on Ice and Strictly Come Dancing. 0.46 of the votes cast were for X Factor, 22% of the public votes favoured Britain's Got Talent, 0.11 voted for Strictly Come Dancing while the remaining votes cast were for Dancing on Ice.

1. What percentage of people voted for Dancing on Ice?
2. How many votes did X Factor receive?

Solution

1. In order to calculate the percentage of people that voted for Dancing on Ice we must calculate the percentage of people that voted for each of the other shows. 0.46 voted for X Factor.

We know $0.46 = \frac{46}{100} = \frac{46}{100} \times \frac{100}{1}\ \% = 46\%$

0.11 of the votes cast were for Strictly Coming Dancing.

We know $0.11 = \frac{11}{100} \times \frac{100}{1}\ \% = 11\%$.

Overall we know that 46 + 22 + 11 = 79% of the votes were for the first three talent shows.

The remainder is therefore equal to 100 – 79 = 21%

As a result we know that 21% of the votes were for Dancing on Ice.

2. X Factor received 0.46 (or 46%) of the vote.

In order to work out what 0.46 of 350,000 is we must convert 0.46 into a fraction

$0.46 = \frac{46}{100} = \frac{23}{50}$

$\Rightarrow \frac{23}{50}$ of $350{,}000 = \frac{23}{50} \times 350{,}000 = 161{,}000$

161,000 people voted for X Factor in this category.

Now you try this.

- Strictly Come Dancing received the fewest votes. Calculate the number of people who voted this show as their favourite talent show.

- Calculate the difference in the number of votes cast for Britain's Got Talent and the number of votes cast for Strictly Come Dancing.

- Work out which show got the second highest number of votes in this category.

Practise your skills

- Practice Sheet N5

Activity Salaries and wages Code N6

This activity links to award learning outcomes 1.1, 1.2, 1.4 and 1.7.

Introduction

Percentages and fractions are part of everyday life. Earlier, you learned how to work with fractions and percentages. In this Activity you will use what you already know about percentages and fractions in order to calculate salaries and wages.

Materials you will need

Calculator

Learning Outcomes

1. Understand the difference between gross and net pay.
2. Calculate wages, salaries and deductions.

Key Learning Points

1. Gross Income
2. Net Income

What do you need to know before you start?

A salary or wage is the money an employer pays to an employee. This could be a weekly wage or a monthly salary. All workers get a pay slip. This pay slip shows the pay the worker earned and any deductions taken off it.

The money that a worker earns is called **gross pay**. This is always reduced by **deductions** and the money that is left is the actual take home pay. This take home pay is called **net pay**.
There are many types of deductions taken from the gross pay. Some of these are optional, such as a trade union subscription. Others are compulsory, such as tax (PAYE), PRSI and the income levy.

Tip: PAYE means: Pay As You Earn.
PRSI means: Pay-related Social Insurance.

Task 1: Calculating net pay

Example

Kevin is a lorry driver. He works a 40-hour week and is paid €8.90 an hour. He gets paid time and a half for overtime.

This week Kevin worked his 40 hours plus 10 hours overtime.

(a) What is Kevin's **gross pay**? _____

Kevin also paid the 2% income levy on his gross income and a further €113.60 in PAYE and PRSI deductions.

(b) How much **income levy** did Kevin pay? _____

(c) What is Kevin's **net pay**? _____

- **Remember:**

  To find a percentage of a number we first convert the percentage to a fraction.

Solution

(a) **Kevin's gross pay is: €489.50**

40 hours at €8.90 per hour

40 x 8.90 = €356

Kevin also worked 10 hours overtime.

Overtime is paid at time and a half i.e. €8.90 + half of €8.90

€8.90 ÷ 2 = €4.45

Overtime rate of pay is €8.90 + €4.45 = €13.35

10 hours at €13.35 per hour is:

10 x 13.35 = €133.50

Kevin's gross pay is his basic pay plus his overtime

€356 + €133.50 = €489.50

(b) **Kevin paid an Income Levy** of **€9.79**

Income Levy of 2% is calculated from Kevin's gross pay.

Gross pay = 489.50

2% = $\frac{2}{100} = \frac{1}{50}$

489.50 x $\frac{1}{50}$ = $\frac{489.5}{50}$ = 9.79

Therefore €9.79 is deduction from Kevin's salary as income levy.

(c) Kevin's **net pay** is what is left when we work out his gross pay minus all the deductions:

€489.50 - €9.79 – €113.60 =

Kevin's net pay = **€366.11**

Task 2: Pay Slips

Example

Katie has a **tax credit** of €80 per week. **This means** that Katie can earn €80 gross pay each week **before** deductions are made. The deductions include PAYE, PRSI and Katie's payment towards her pension.

Complete Katie's weekly payslip below.

Name: Katie Smith **Staff Number:** 1234 **RSI Number:** 1000001A **Date:** 25th June 2011	**Deductions**
Pay **Basic Pay:** 38 x €8.75 **Overtime:**1.5 hours x double time **Overtime:** 3 hours x time and a half **Gross Pay =**	**Income Levy 2%** **PAYE 20%** **PRSI 6%** **Pension** €4.50 **Total Deductions** **Net Pay**

Solution

Katie's Gross Pay:

38 hours at €8.75 = **€332.50**

Double time rate of pay = 2(€8.75) = €17.50

Time and a half rate of pay = (€8.75) ÷ 2 + €8.75 = €13.13

2 hours at €17.50 = **€35**

3 hours at €13.13 = **€39.39**

Gross pay = **€332.50 + €35 + €39.39 = €406.89**

Income Levy:

2% of gross income

2% = $\frac{2}{100} = \frac{1}{50}$

406.89 x $\frac{1}{50}$ = $\frac{406.89}{50}$ = 8.14

Income Levy = **€8.14**

PAYE:

PAYE is calculated on gross income – tax credit

€406.89 - €80 = €326.89

PAYE rate is 20%

20% = $\frac{20}{100} = \frac{1}{5}$

326.89 x $\frac{1}{5}$ = $\frac{326.89}{5}$ = 63.378

PAYE = **€63.38**

PRSI:

PRSI is calculated on gross income – tax credit

€406.89 - €80 = €326.89

PRSI rate is 6%

6% = $\frac{6}{100} = \frac{3}{50}$

326.89 x $\frac{3}{50}$ = $\frac{980.67}{50}$ = 19.6134

PRSI = **€19.61**

Total Deductions:

Income Levy = **€8.14**

PAYE = **€63.38**

PRSI = **€19.61**

Pension = **€4.50**

€8.14 + €63.38 + €19.61 + €4.5 = €95.63

Net Pay:

Gross Pay – Total Deductions **= €406.89 - €95.63**

Net Pay = €311.26

Now you try this.

- A cosmetics company sales representative earns a basic salary of €1,240 per month. She also earns 15% on any sales that she completes. In May she sells €3,200 worth of cosmetics. **What is her monthly gross pay?**

- Mr. Murphy works in a busy city centre bar. He earns €9.80 per hour and works a 39 hour week. This week he worked 5 hours overtime which are paid at time and a half. Mr. Murphy has a tax credit of €111. He pays the 2% Income Levy, 20% PAYE and 6% PRSI. He also contributes €3.30 to his pension fund. **Create a payslip for Mr. Murphy**. You may leave his other employee details blank.

Practise your skills

- Practice Sheet N6

Activity Profit & Loss Code N7

This activity links to award learning outcomes 1.1, 1.2, 1.4 and 1.7.

Introduction

Earlier you learned how to work with payslips, working out wages, salaries and taxes for employees. In this activity you will look at how people who are **self employed** get paid. Many trades people, shopkeepers, writers, artists, doctors, dressmakers, lorry drivers and other workers are **self-employed**. They rely on making a profit in order to pay their own wages. So they need to be able to keep track of how much money they make through their work, and how much they spend on it. To do this, they keep a record called a 'profit and loss account'. This activity will help you to be able to calculate profit and loss.

Materials you will need

1. Calculator

Learning Outcomes

Understand the concept of profit and loss.

Calculate profit and loss.

Key Learning Points

1. Profit
2. Loss

What do you need to know before you start?

Maths

Profit is money left over after all the expenses are paid. A **loss** is when the expenses are greater than the money coming in (income).

Here is **an example of a profit and loss account** for a self employed shoemaker for the financial year 2009/2010.

Sample profit and loss account

Total Payments (money received)		**€21,560**
Cost of Materials		**€8,640**
	Gross profit:	**€12,920**
Operating Costs		
Rent		**€2,800**
ESB		**€1,550**
Equipment		**€ 620**
		€4,970
	Operating profit:	**€7,950**

The shoemaker's profit for that period was €7,950. The shoemaker might decide to pay himself all of that as a salary, or to put some aside to invest back into the business.

Task 1: Calculating net pay

Example

Kevin is a self employed electrician. Last year he received €65,542 in payments for work done. He recorded his costs for the year as follows:

Workshop Rent:	€3,200
Van Expenses:	€2,100
Phone Bill:	€385
Advertising (radio):	€590
Insurance:	€678
ESB:	€340
Equipment:	€12,978

Try this before looking at the solution:

a. Create Kevin's profit and loss account.
b. What was Kevin's salary last year if he set aside 12% for future reinvestment?

Solution

(a) Kevin's profit and loss account:

Total Payments (money received)	**€65,542**
Cost of Materials	**€12,978**
Gross profit:	**€52,564**

Operating Costs

Rent	€3200
Phone	€385
Insurance	€678
Van	€2100
Advertising	€590
ESB	€340
	€ 7,293

Operating profit:€45,271

Therefore Kevin's profit is €45,271.

(b) Kevin's salary:

Kevin's profit = €45,271. He reinvested 12% of this profit.

$$12\% = \frac{12}{100} = \frac{3}{25}$$

$$\frac{3}{25} x \frac{45271}{1} = \frac{135813}{25} = 5{,}432.52$$

- Therefore Kevin reinvested €5,432.52, leaving him with a salary of **€39,838.48.**

Task 2: Cost of a Job

Example

Tony is a gardener. He charges €8.25 an hour for his services. At the moment he is trying to get a job re-seeding a lawn for Mr. and Mrs. Murphy. He has told them that the job will take about 8 hours and that it will need 6 bags of grass-seed (€24.60 each) and 2 bags of fertiliser (€18.75 each).

(a) Tony needs to give Mr. and Mrs. Murphy the price of this job before he can secure the job.

What is the estimated cost of this job?

(b) When Tony is unloading the bags of grass seed he rips three of them and loses the seed. He must now buy another three bags. It was his own mistake, so he can't charge the customers for it. Mr. and Mrs. Murphy pay Tony €250 for the job.

Has Tony made a profit or loss on this job? How much of a profit or loss?

Solution

(a) **Costs:**

Materials:

Grass Seed	6 x €24.60	= €147.60
Fertiliser	2 x €18.75	= € 37.50
	€185.51	

Labour

8 hours at €8.25 an hour = €66.00

Total Costs: €185.51 + 66 = €251.10

(b) **Profit or loss?**

Tony now has to buy another three bags of grass seed.

3 x €24.60 = €73.80

Mr. and Mrs. Murphy have paid Tony €250.

Tony's actual costs end up being total costs plus the cost of the three extra bags: €185.51 + 73.80 = €259.31

Therefore Tony makes a loss of €9.31, however he does not get any wages for this job now so in total Tony makes a loss of €9.31 + €66 = €75.31.

Now you try this.

Draw up a profit and loss account for 2009 for Sean, a self employed lorry driver:

Total payments:	€68,985
Road Tax & Insurance:	€ 4,300
Maintenance & Diesel:	€ 4,286
Advertising:	€ 875

- **What is Sean's salary?** Assume it is the same as the net profit.

- Mrs. Smyth runs a corner shop. Her total sales last year were €48,976 and the total cost of goods last year came to €31,486.55. **What was Mrs. Smyth's gross profit?**

- Sheila is an interior decorator. She charges €9.10 per hour. She is painting a room for a customer. It takes 4.5 hours to paint the room and 3.8 buckets of paint . The paint costs €25.60 per bucket. **What is the total cost of this job?**

Practise your skills

- Practice Sheet N7

Activity Tourist Destinations Code N8

This activity links to award learning outcomes 1.1, and 1.2

Introduction

There are many areas in Ireland which are well-known as tourist destinations: for example, areas in Donegal, Kerry and Clare.

Materials you will need

- Map of Ireland with popular tourist destinations highlighted
- Images of popular tourist attractions in Dublin
- Calculator
- Pen & Paper

Learning Outcomes

1. Understand the concept of ratio.
2. Convert ratios to fractions.

Key Learning Points

1. Ratio
2. Conversion

What do you need to know before you start?

Maths

A ratio is simply a comparison of two numbers. We generally separate the two numbers in the ratio with a colon: For example, 5:10.

In a group of 30 people in a room, 10 people like to watch Coronation Street, 15 people prefer Fair City and 5 people watch Emmerdale. So the ratio of Fair City fans to Coronation Street fans is fifteen to ten. In maths symbols, we write it like this, using the colon: 15:10.

There is **a strong relationship between ratios and fractions.** Just like fractions, if the two numbers in our ratio have a common factor then we can simplify them. So, in our example, we can simplify the ratio of Fair City fans to Coronation Street fans to 3:2. For every two Coronation Street fans in the room there are three Fair City fans.

Ratios are simply another way of writing fractions. In order to convert a ratio to a fraction the first number in the ratio acts as the **numerator** and the second number acts as the **denominator.**

Therefore to express the ratio of Fair City fans to Coronation Street fans in the room we say that it is 3:2 **=** $\frac{3}{2}$**.** For every two Coronation Street fans in the room there are three Fair City fans.

Tourist Attractions

- Popular tourist destinations in Ireland.
- Popular attractions in Ireland.

How can you find this out?

- Visit the Tourism Ireland website.
- Visit a local tourist office.
- Ask friends and family member.

Task 1 Understanding Ratios

Example

Five popular tourist attractions in Dublin are the Guinness Storehouse, St. Stephens Green, Dublin Castle, Natural History Museum and Dublin Zoo. A survey was carried out among 100 people who visited all five attractions in 2009 and 35 people said Dublin Zoo was their favourite attraction, 20 people preferred the Guinness Storehouse, 15 people said St. Stephens Green was their favourite attraction, 5 people preferred the Natural History Museum while the remaining chose Dublin Castle as their favourite attraction.

Use that information write out the following ratios:

(1) The ratio of people who prefer Dublin Zoo to those who prefer St. Stephens Green.

(2) The ratio of people who prefer Dublin Castle to those who prefer the Guinness Storehouse

Task 1 Understanding Ratios

Solution

(1) **The ratio of people who prefer Dublin Zoo to those who prefer St. Stephens Green**

35 people preferred Dublin Zoo.

15 people preferred St. Stephen's Green.

Therefore the ratio of people who prefer Dublin Zoo to those who prefer St. Stephen's Green is 35:15.

We see that both these figures have a common factor of 5 and so we can simplify this to 7:3.

That means for every 3 people who prefer St. Stephen's Green 7 prefer Dublin Zoo.

(2) The ratio of people who prefer Dublin Castle to those who prefer the Guinness Storehouse

We know that 20 people prefer the Guinness Storehouse while we must calculate the number of people who prefer Dublin Castle.

In total 100 people were surveyed so from this we must take away the number of people who prefer the other five attractions.

Therefore the number of people who prefer Dublin Castle:

100 – 35 – 20 -15 – 5 = 25

⇒ 25 people prefer Dublin Castle.

The ratio of people who prefer Dublin Castle to those who prefer the Guinness Storehouse is 25:20.

5 is a common factor and so we can simplify this ratio:

25:20 = 5:4

Now you try this.

- Write down the ratio of people who prefer the History Museum to those who prefer St. Stephen's Green. Simplify this ratio if possible.

Did you remember?

- To simplify we take out any common factors.
- The place that comes first in the word problem comes first in the ratio.

Task 2: Converting Ratios to Fractions

Example

Four popular tourist destinations in Ireland are (1) the Ring of Kerry, (2) the Cliffs of Moher, (3) Kilkenny Castle and (4) the Giant's Causeway. A group of one hundred students were asked which of these destinations they had visited. 24 had visited the Cliffs of Moher only, 12 had been to the Ring of Kerry only, 20 had visited Kilkenny Castle only and 4 had been to the Giant's Causeway only.

However, 22 said they had been to both the Giant's Causeway and Kilkenny Castle. The remaining 18 students had not yet been to even one of these destinations.

- **What is the ratio of people who have visited 2 destinations to the people who have visited none of the listed destinations?**

- **For every one person that visited the Ring of Kerry how many people visited the the Cliffs of Moher?**

Solutions

- **What is the ratio of people who have visited 2 destinations to the people who have visited none of the listed destinations?**

We know that 22 people visited two destinations while 18 people have yet to visit any.
So the ratio we are looking for is 22:18
2 is a common factor of both 22 and 18 and so our ratio can be simplified down to 11:9

- **For every one person that visited the Ring of Kerry how many people visited the the Cliffs of Moher?**

In order to do this question we must first find the ratio of people who visited the Cliffs of Moher to the people who visited the Ring of Kerry.
Ratio = 24:12 = 2:1
When we write this as a fraction we get $\frac{2}{1}$ which tells us that for every person that visited the Ring of Kerry, 2 people visited the Cliffs of Moher.

Now you try this.

- Find the ratio of the number of people who visited the Giant's Causeway to the number of people who visited the Cliffs of Moher.

- For every person that visited the Giant's Causeway how many people visited the Ring of Kerry?

Practise your skills

- Practice Sheet N8
- If there is a talent show on TV, such as X Factor, do a survey of your centre to find out which contestant is the favourite. Find the ratio between the supporters of different acts.

Activity **Route 66** **Code N9**

This activity links to award learning outcomes 1.1, 1.2 and 1.3

Introduction

Route 66 is one of the original American Highways and was opened on the 11th November 1926. It runs from Chicago to Santa Monica. Every year a large number of tourists from around the world drive this route. It is so popular both among Americans and people visiting the States that many songs have been written about it. This activity will help you to be able to convert currency – for example, change Euro to US dollars.

Materials you will need

- A map of America highlighting Route 66
- Current exchange rates (available on any bank's website)
- Pen & Paper
- Dollars & Euro
- Calculator

Learning Outcomes

1. Understand the idea of currency exchange and conversion.
2. Convert dollars to euro and euro to dollars.

Key Learning Points

1. Currency exchange
2. Conversion rates

What do you need to know before you start?

Maths

Currency exchange is important in all walks of life. For example, if Irish companies weren't able to exchange currency with other countries it would be impossible to import foreign goods such as Ipods, cars, oil etc. Exchange rates allow people to convert their money from one currency to another. For example, if a local electrical store wishes to buy 100 iPods from Sony Headquarters in Japan then they will need to convert their Euros to Yen in order to pay for them. Yen is the Japanese currency. **The exchange rate between two currencies tells us how much one currency is worth in terms of the other.**

Look at the exchange rate chart below. The AIB Bank displayed these exchange rates in all their branches in November 2010:

Currency	**We Buy**	**We Sell**
Sterling	0.8583	0.8347
US Dollars	1.378	1.34
Australian Dollars	1.401	1.3624

To understand what the exchange rate means in the real world, you need to **round off** some of the figures. For example, this chart tells us that for every 1 Euro you have, the bank will 'sell' you 0.83 Pounds sterling. (Sterling is also called Great British Pounds, or GBP). Also it tells us that if you want to change sterling back to Euro the bank will 'buy' 86 pence off you in exchange for one Euro.

This is how you would **use the exchange rates on the chart to convert €50 into pounds sterling:**

For every €1 you would get 83 pence. So, multiply 50 by 83 in order to see how much €50 is worth in sterling.

Using these exchange rates we get 50 x 0.83 = £41.50 sterling.

If you want to change £20 back into euro then you must divide £20 by 0.86. That will show how many '86 pences' are in £20: that is how many euro you would be due.

Using these exchange rates you would get 20 ÷ 0.86 = €23.26

A general rule of thumb for converting currency:

- If you are changing euro to foreign currency in Ireland, multiply your amount by the '**we sell'** rate.
- If you are changing foreign currency to euro in Ireland, divide your amount by the '**we buy'** rate.

Route 66

For this activity, you should know

- current exchange rates,
- the currency used in different countries.

How can you find this out?

- Use the internet
- Visit your local bank
- Ask friends/family members

Task 1: Understanding Exchange Rates

Example

Use the following information on exchange rates to answer the questions below.

Currency	We Buy	We Sell
Latvian Lat	0.6996	0.7194
US Dollars	1.378	1.34
Australian Dollars	1.401	1.3624
Thai Baht	39.9915	41.1271

(a) **How many Thai Baht (rounded to two decimal places) would I get for €1?**

(b) **How many US dollars (rounded to two decimal places) would I need in order to get one euro?**

Solution

(a) **How many Thai Baht (rounded to two decimal places) would I get for €1?**
(b) **The table shows that the bank sells 41.1271 Thai Baht. Therefore for every euro I have I would get 41.13 Thai Baht.**

(c) **How many US Dollars (rounded to two decimal places) would I need in order to get one Euro?**

The table shows that the bank buys 1.378 US Dollars.

Therefore in order to get one euro I would need to have 1.39 US dollars.

Now you try this.

How many Latvian Lat (rounded to two decimal places) would I get for €1?

_____

How many Australian dollars (rounded to two decimal places) would I need in order to get €1?

_____

Did you remember?

- To round off your numbers
- If exchanging euro to foreign currency we **multiply** by the 'We Buy' rate.
- If exchanging foreign currency to euro we **divide** by the 'We Sell' rate.

Task 2: Route 66

Example

A group of friends want to travel route 66 during their summer holidays. They want to buy the car before they leave Ireland but they will buy it from an American dealer. They see a used car online and it is priced at **$2,500 dollars**. When they check the exchange rate at the bank this is the information they see:

Currency	We Sell	We Buy
US Dollars	1.378	1.34

What is the price of the car in euro?

_____

Solution

In order to solve this problem the friends must convert US dollars to euro. Every $1.38 is worth €1.

2500 ÷ 1.38 = €1,811.5942

The car would cost the friends €1,811.59

Another example using that exchange rate table above:

The friends decide to buy this car. When they split the cost between them they each still have **€750**. How many dollars will they get for this if they exchange them on the same day?

Solution

We know from the table that for every euro they will get US$1.34.

Therefore in order to find out how much their spending money is worth in dollars we must multiply 750 by 1.34. 750 x 1.34 = 1,005.

So, each of the friends has US$1,005.

Now you try this.

- While in the US, one of the friends spots a jacket in a store. It is priced at $40. However before he left Ireland he saw a jacket he liked for €30 in a sports shop at home. The exchange rate on the day was:

Currency	We Sell	We Buy
US Dollars	1.378	1.34

Which jacket is cheaper for this person to buy: the one in Ireland or the one in US?

_____

- One of the friends ran out of money. His family sent him €200. At the time the exchange rate was:

Currency	We Sell	We Buy
US Dollars	1.364	1.20

How many extra dollars did he now have?

_____

Practise your skills

- Practice Sheet N9

Activity **Planets** **Code N10**

This activity links to award learning outcomes 1.1, 1.2 and 1.4.

Introduction

When we look at the distance between the planets in the solar system the numbers we are dealing with can be very large. For example the distance from the Earth to Mars is approximately 78,300,000 kilometres. Such big figures often make it very hard to work with distances. How, we can re-write these numbers to make it easier to work with them to solve problems.

Materials you will need

- Images of different planets in our solar system
- Pen and paper
- Calculator

Learning Outcomes

1. Understand the idea of scientific notation.
2. Convert numbers from standard form to scientific notation.
3. Convert numbers from scientific notation to standard form.

Key Learning Points

1. Scientific notation.

What do you need to know before you start?

Maths

Scientific notation allows us to handle very large or very small numbers easily. It requires us to first have a good understanding of decimals and indices (See Activity N13).

In order to write a number in scientific notation we write it as $\mathbf{a \times 10^n}$ where a is between 1 and 10 ($1 \leq a < 10$) and n is an integer.

For example earlier we said that Mars was 78,300,000 kilometres from Earth. In scientific notation $78{,}300{,}000 = 7.83 \times 10^7$.
That is because we have to multiply 7.83 by 10 seven times in order to get 78,300,000.

10^7 means ten to the power of seven. As you can see the power to which ten is raised (seven) is equal to the number of places we have shifted the decimal point in order to give the number in its longer form.
We can also write very small numbers using scientific notation.
For example: We can express the weight of an object in kilograms or pounds. We know that one gram is equal to **0.0022** of a pound. We can also write this very small number as $\mathbf{2.2 \times 10^{-3}}$.

Again the power to which ten is raised shows the number of places we must shift the decimal point in order to get back to our original number. However, this time the point is moving in the opposite direction.

Finally it is important to note that when the power to which ten is raised is a positive number the decimal point moves to the right but if the power is negative it moves to the left.

Planets

It would be useful to know:

- The different planets in the solar system
- Where the planets are located in our solar system

How can you find this out?

- Use the internet.
- Ask friends/family members.
- Look in second level science books.

Task 1: Distance from the Earth to different planets

Example

The following table outlines the distance between the Earth and other planets in the solar system:

Planet	**Distance to Earth (in Km)**
Saturn	1.321×10^8
Mercury	7.7×10^7
Pluto	5.913×10^9

Write the distance between Earth and Saturn and between Earth and Mercury in standard form.

Distance between Earth and Saturn: _____

Distance between Earth and Mercury: _____

Solution

In scientific notation the distance between Earth and Saturn is 1.321×10^8. This means to rewrite this number in standard form we must move the decimal place 8 places to the right.

So the distance between Earth and Saturn is 132,100,000 kilometres.

The distance between Mercury and Earth is written as 7.7×10^7.

This tells us that the decimal point must move the decimal point 7 places to the right.

$7.7 \times 10^7 =$ 77,000,000 kilometres.

So the distance between Earth and Mercury is 77,000,000 kilometres.

Now you try this.

Write the distance between Pluto and Earth in standard form.

Did you remember?

- The power to which ten is raised tells us how many places we must move the decimal point.
- A positive power means we move the decimal point to the right.

Task 2: The Sun, the Moon and the Earth

The average distance between the Sun and the Earth is 92,900,000 kilometres. The approximate distance between the Sun and the Moon is 150,000,000 kilometres. **Write these distances using scientific notation.**

Solution

The distance between the Sun and the Earth is 92,900,000.

When writing this number in scientific form we must make sure that the number is less than 10.

Therefore $92,900,000 = 9.29 \times 10^7$

Similarly we must make sure that when converting the distance between the sun and the moon the number which we multiply by 10^n is less than 10.

Therefore $150,000,000 = 1.5 \times 10^8$

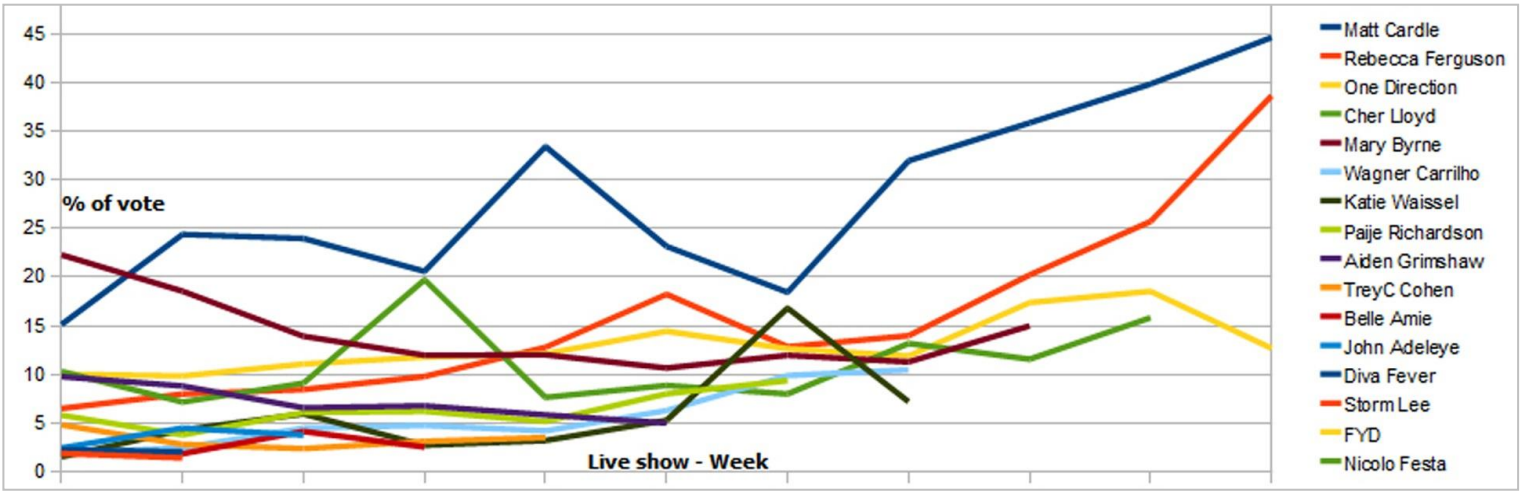
Now you try this.

The distance between the Earth and the Moon is 384,400 kilometres. The Moon is approximately 42,100,000 kilometres from the planet Mars. **Write both these distances using scientific notation.**

Practise your skills

- Practice Sheet N10

Activity **X Factor Voting** **Code N11**



This activity links to award learning outcomes 1.1 and 1.3.

Introduction

The X Factor is a television talent show and each week members of the public must phone in to vote to save their favourite act. The results are often extremely close.

Materials you will need

- Numberline
- Calculator
- Pen & Paper

Learning Outcomes

1. Understand the concept of rounding off.
2. Round off decimals to two decimal places.

Key Learning Points

1. Decimals
2. Round Off

What do you need to know before you start?

Maths

There are times when we divide one integer by another and our answer can look strange or complicated. For example, if we divide 4 by 7 using a calculator we will get an answer of 0.571428571. This is an exact decimal figure. It is too complicated to deal with it in everyday real life. As a result we often round off this decimal to make it more manageable.

How to round off to two decimal places

Just like when we round off whole numbers (which you did in Level 3), we can round off decimals.

Example:

To round off 0.571428571 to two decimal places:

- Look at the third digit after the decimal place (1).
- Since this number is less than 5, the number before it (7) remains unchanged.
- Therefore, the number 0.571428571 becomes **0.57** when rounded off to two decimal places.

Now let's round off **1.43925672** to two decimal places and after that we'll round it off to three decimal places.

Rounding 1.43925672 to two decimal places:

The third digit after the decimal point is 9. Since 9 is greater than 5, we increase the previous number by 1 and that gives us **1.44.**

This makes sense since 1.439 is closer to 1.44 than to1.43.

Rounding 1.43925672 to three decimal places:

Look at the fourth digit after the decimal place. It is 2. Since 2 is less than 5, the previous digit remains the same. That means we get **1.439**.

Again, this makes sense since 1.4392 is closer to 1.439 than to 1.440.

As a rule of thumb:

When the number is greater than 5 the previous digit increases by 1.

When the number is less than 5 the previous digit does not change.

The X Factor

For this activity it would be useful to know

- how voting works for the X Factor
- the X Factor contestants in 2009
- how the judges decide who to eliminate.

How can you find this out?

- Visit the X Factor website
- Ask friends
- Ask family members
- Look on You Tube for clips from the show.

Task 1: Rounding Off Decimal Places

Example

Look at the numbers in the box below. Round off the five numbers in the top line to **one** decimal place. Round off the five numbers in the bottom line to **two** decimal places.

1.46	2.53	6.751	11.348	15.671
2.871	3.789	5.4812	13.446	22.356

Rounding Off Decimal Places

Solution

Top row: numbers rounded off to one decimal place

1.46 = 1.5

2.53 = 2.5

6.751 = 6.8

11.348 = 11.3

15.671 = 15.7

Bottom row: numbers rounded off to two decimal places

2.871 = 2.87

3.789 = 3.79

5.4812 = 5.48

13.446 = 13.45

22.356 = 22.36

Now you try this.

11.348 rounded to one decimal place _____

15.671 rounded to one decimal place _____

13.446 rounded to two decimal places _____

22.356 rounded to two decimal places _____

Did you remember?

- To round to one decimal place means we have one digit after the decimal point.
- To round to two decimal places means we have two digits after the decimal point.
- When rounding off to one decimal place we look at the value of the second digit after the decimal point.
- When rounding off to two decimal places we look at the value of the third digit after the decimal point.

Task 2: Rounding off and calculating differences

Example

On the 21st November 2009 Olly Murs and Jedward were in the bottom two when the X Factor results were read out. Both then had to perform in the sing off and hope that the judges saved them. Olly sang *Wonderful Tonight* by Elton John. Jedward sang *No Matter What* by Boyzone. 3 of the 4 judges voted to save Olly and so Jedward left the competition.

However results from the viewers' vote showed that if the vote had gone to deadlock Olly was the contestant with the fewest votes. The result was the closest in X Factor history: Olly received 18.6879% of the vote while Jedward received 18.7935% of the vote.

Round off both contestants' percentage of the vote to one decimal place.

Olly: _____

Jedward: _____

Solution

Olly: We know that Olly received 18.6879% of the vote.
In order to round this off to one decimal place we must consider the second digit after the decimal place: 18.6**8**79
Because 8 is greater than 5 we round up: so Olly got 18.7% of the public vote.
Jedward: We know that Jedward received 18.7935% of the vote.
In order to round this off to one decimal place we must look at the second digit after the decimal place: 18.7**9**35
Because 9 is greater than 5 we round up.
Therefore Jedward got 18.8% of the public vote.

Now, using these rounded off decimals **calculate the approximate difference** between the percentage of people who voted for Olly and the percentage of people who voted for Jedward. Try to work it out yourself before reading the solution below.

Solution

18.7% voted for Olly while 18.8% voted for Jedward.
We now must subtract these two decimals to find out the difference between the two.

$$\begin{array}{r} 18.8 \\ -\ 18.7 \\ \hline 0.1 \end{array}$$

0.1% more people voted for Jedward than voted for Olly.

Now you try this.

Round off both contestants' percentage votes to **two** decimal places.

Use the figures with 2 decimal places to calculate the approximate difference in the percentage of people who voted for Jedward and Olly.

In the final, Joe Mc Elderry received 61.5673% of the vote, Olly Murs received 34.831% and Stacey Solomon received 3.6017% of the vote. Round each of their final percentages to one decimal place.

Joe: _____

Olly: _____

Stacey: _____

Practise your skills

- Practice sheet N11

Activity **Antique Roadtrip** **Code N12**

This activity links to award learning outcomes 1.1 and 1.3.

Introduction

The Antique Roadtrip is a British reality television show. Eight antique experts travel across Britain looking for items that may be worth a lot of money for the person who wants to sell them. They choose items to sell at auction. At an auction, people bid for the item: the person bidding the highest price gets the item. Before the experts put an antique item up for auction, they estimate how much they think the item will sell for. Most of the time their estimate is pretty accurate but sometimes they are way off the mark!

Materials you will need

- Clips from the Antique Road Trip if they are available.
- Different items of food/clothing in order for you to estimate the price
- Pen & Paper

Learning Outcomes

1. Understand the concept of percentage error.
2. Calculate percentage error.

Key Learning Points

1. Percentage error

What do you need to know before you start?

Maths

Percentage error is about the difference between an estimated value and the accurate value expressed as a percentage.

For example in the TV show 'The Apprentice' in 2010, Team Elev8 asked members of the public to guess how many nails were in a jar. The first person guessed that there were 456 nails in the jar. In fact there were 535.

In order to see how close this guess was we would subtract 456 from 535.

535 – 456 = 79. So we know that he was 79 nails off the correct answer. Another way of saying this is that 79 was his margin of error.

In order to express this margin of error as a percentage we would have to calculate:

$$\frac{Error}{Correct\ Value} \times \frac{100}{1}\%$$

The error over the correct value will give me the margin of error in fraction form. We know from studying percentages that in order to convert this fraction to a percentage we must always multiply by $\frac{100}{1}\%$.

Antique Roadtrip

- Estimated value of certain antiques

How can you find this out?

- Use the internet.
- Visit the Antiques Roadtrip website.
- Ask friends/family members.
- Inquire in local antique dealers.

Task 1: Antique Road trip

Example

In one episode of Antiques Roadtrip one of the experts predicted that a Molin painting would sell for £18,000. Another expert believed it would sell for £26,500. When it went to auction it sold for £25,000.

Calculate the percentage error for both the experts.

Solution

Expert 1

Estimate = 18000

Difference = 25,000 – 18,000 = 7,000

Percentage Error = $\frac{7000}{25000} \times \frac{100}{1}\%$

Percentage Error = 28%

Expert 2

Estimate = 26,500

Difference = 26,500 – 25,000 = 1,500

Percentage Error = $\frac{1500}{25000} \times \frac{100}{1}$

Percentage Error = 6%

Now you try this.

On another episode of the show a person was looking to auction a Grandfather clock. Again two experts estimated what the clock would fetch at auction. The first expert guessed it would make €4,000 while the second expert thought it would only get £2,000. In the end the clock sold for £2,500. **Calculate the percentage error for each expert.**

Did you remember?

- To calculate the difference between the estimate and the actual price first.
- In order to find the percentage error we must calculate

$$\frac{Error}{Correct\ Value} \times \frac{100}{1}\%$$

Practise your skills

- Practice sheet N12

- Try this for your next class: Everyone in the group should bring in some items from home, for example food, clothing and so on. You must know the price of the item you bring in. Then everybody else in your group must estimate how much the item cost. When you tell them the correct price **everyone must work out their percentage error**. Whoever had the closest estimate wins that round. After a certain number of rounds add up everyone's total score to see who wins.

Activity **Indices** **Code N13**

This activity links to award learning outcomes 1.1, 1.4 and 1.5.

Introduction

Writing numbers using index (or power) notation is the shorthand for repeated multiplication. For example, 3 x 3 is 3 multiplied by itself twice. We can write this as 3^2, and we would say 'three squared' or 'three to the power of 2'. In 3^2, 3 is called the **base number** and 2 is called a **power** or **index.** The plural of index is **indices**.

A **base number** is the number that is being multiplied.
The **power** tells us how many times we multiply the base number by itself.

Materials you will need

- Calculator

Learning Outcomes

1. Understand the laws of indices.
2. Apply to laws of indices.

Key Learning Points

1. Indices

What do you need to know before you start?

Maths

There are 8 **rules of indices** in total. In this section we are going to look at five of these.

In order to apply these rules you need to have completed some of the algebra activities in Unit 3.

$a^2 = a \times a$

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

$a^{10} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

Rule 1

In Unit 3 (Algebra) you learned that we can use letters to represent numbers. For the rules of indices we will let a be any number. That is, we don't know what a is but we can still use it as though it is a number.

Example 1.1

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

Let's multiply a^3 and a^4 $(a \times a \times a) \times (a \times a \times a \times)$

We now have $a \times a \times a \times a \times a \times a \times a = a^7$

Example 1.2

$a^2 = a \times a$ $a^6 = a \times a \times a \times a \times a \times a$

Let's multiply a^2 and a^6: $(a \times a) \times (a \times a \times a \times a a \times a)$

We now have $a \times a \times a \times a \times a \times a \times a \times a = a^8$

Can you spot any pattern for multiplication?

When we are working with larger powers such as $a^{27} \times a^{19}$ it is no longer practical to list out all the $a's$ and count them.

From Example 1.1 we know that $a^3 \times a^4 = a^7$

So, if we added the powers we would have the correct answer.

From Example 1.2 we know that $a^2 \times a^6 = a^8$

It is also clear from this example that if we added the powers we would have the correct answer.

Therefore for $a^{27} \times a^{19}$, instead of listing out all the $a's$ we can add the powers.

$a^{27} \times a^{19} = a^{27+19} = a^{46}$

Rule 1: $a^m \times a^n = a^{m+n}$

As you can see from the above examples rule 1 applies when you are multiplying the numbers with the same base number.

Example 1.3

Using the rules of indices, simplify the following: $4^{13} \times 4^{16}$

$$4^{13} \times 4^{16} = 4^{13+16} = 4^{29}$$

Rule 2

Example 2.1

$a^{13} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

$a^{4} = a \times a \times a \times a$

Let's divide a^{13} by a^{4}:

$$(a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a) \div (a \times a \times a \times a)$$

$$= \frac{a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$$

$$= a \times a \times a \times a \times a \times a \times a \times a \times a$$

which is equal to a^{9}

Example 2.2

$a^{6} = a \times a \times a \times a \times a \times a$ $\quad$ $a^{2} = a \times a$

Let's divide a^{6} by a^{2}: $\quad$ $(a \times a \times a \times a \times a \times a) \div (a \times a)$

$$= \frac{a \times a \times a \times a \times a \times a}{a \times a}$$

$$= a \times a \times a \times a$$

which is equal to a^{4}

Can you spot any pattern for division?

Like multiplication when we are working with larger powers it is not practical to list out all the a's and count them.

From Example 2.1 we know that $a^{13} \div a^{4} = a^{9}$

It is clear from this example that if we subtract the powers we would have the correct answer.

$a^{13} \div a^{4} = a^{13-4} = a^{9}$

From Example 2.2 we know that $a^6 \div a^2 = a^4$

It is also clear from this example that if we subtract the powers we would have the correct answer.

Therefore for $a^{27} \times a^{19}$ instead of listing out all the a's we can subtract the powers.

$a^{27} \times a^{19} = a^{27-19} = a^8$

Rule 2: $a^m \div a^n = a^{m-n}$

As you can see from the above examples rule 2 applies when you are dividing numbers with the same base number.

Example 2.3

Using the rules of indices simplify the following $6^{115} \div 6^{48}$

$$6^{115} \div 6^{48} = 6^{115-48} = 6^{67}$$

Rule 3

Example 3.1

$a^4 = a \times a \times a \times a$

Therefore $(a^4)^2 = (a \times a \times a \times a)(a \times a \times a \times a)$

$= a \times a \times a \times a \times a \times a \times a \times a$

$= a^8$

Example 3.2

$a^5 = = a \times a \times a \times a \times a$

$(a^5)^4 =$

$(a \times a \times a \times a \times a)(a \times a \times a \times a \times a)(a \times a \times a \times a \times a)(a \times a \times a \times a \times a)$

$= a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

$= a^{20}$

Can you spot any pattern?

From **Example 3.1** we know that $(a^4)^2 = a^8$

It is clear from this example that if we multiply the powers we would have the correct answer.

$(a^4)^2 = (a^{4x2}) = a^8$

From **Example 3.2** we know that $(a^5)^4 = a^{20}$

It is also clear from this example that if we multiply the powers we would have the correct answer.

$(a^5)^4 = (a^{5x4}) = a^{20}$

Therefore for $(a^{27})^{13}$ instead of listing out all the $a's$ we can multiply the powers.

$(a^{27})^{13} = (a^{27x13}) = a^{351}$

Rule 3: $= (a^m)^n = (a^{mxn})$

As you can see from the above examples rule 3 applies when you have two powers being applied to the same base number.

Example 3.3

Using the rules of indices simplify the following $(3^5)^{18}$

$$(3^5)^{18} = (3^{5\,x\,18}) = (3^{90})$$

Rule 4

Let b be another unknown number.

$ab = a \times b$

$(ab)^2 = (a \times b)^2 = (a \times b)(a \times b) = (a \times b \times a \times b)$

$(ab)^3 = (a \times b)^3 = (a \times b)(a \times b)(a \times b) = (a \times b \times a \times b \times a \times b)$

Example 4.1

$(ab)^2 = (a \times b)(a \times b) = (a \times b \times a \times b)$

$= (a \times a \times b \times b)$

$= (a^2 \times b^2) = a^2 \times b^2$

Example 4.2

$(ab)^7 = (a \times b)(a \times b)\ (a \times b)(a \times b)\ (a \times b)(a \times b)\ (a \times b)$

$= (a \times b \times a \times b \times a \times b \times a \times b \times a \times b \times a \times b)$

$= (a \times a \times a \times a \times a \times a \times a \times b \times b \times b \times b \times b \times b \times b)$

$= (a^7 \times b^7) = a^7 \times b^7$

Can you spot any pattern from the examples above?

Discuss this with your group. See if together you can spot the pattern or rule, then read the explanation below.

Spotting the pattern or rule:

From **Example 4.1** we know that $(ab)^2 = a^2 \times b^2$.

It is clear from this example that if we apply a power to a product each factor can be raised to the power.

$(ab)^2 = a^2 \times b^2$.

From **Example 4.2** we know that $(ab)^7 = a^7 \times b^7$

It is clear from this example that if we apply a power to a product each factor can be raised to the power. ('Factor' means the part that is being multiplied – that is, each number)

$(ab)^7 = a^7 \times b^7$

Rule 4: $(ab)^m = a^m \times b^m$

As you can see from the above examples rule 4 can be applied when you have one power being applied to two or more numbers which are being multiplied.

Example 4.3

Using the rules of indices simplify the following $(3\times a)^{18}$

$(3\times a)^{18} = (3^{18}\times a^{18})$

Rule 5

What happens when you have a^0?

Let's explore this idea using rule 2.

Example 5.1

$$(a)^4 \div (a)^4 = \frac{a\times a\times a\times a}{a\times a\times a\times a}$$

From algebra we know that **any number divided by itself is 1**. So here we are left with

$$(a)^4 \div (a)^4 = \frac{a\times a\times a\times a}{a\times a\times a\times a} = 1$$

From rule 2 we also know that $(a)^4 \div (a)^4 = (a)^{4-4} = (a)^0$

Therefore, $(a)^4 \div (a)^4 = \frac{a\times a\times a\times a}{a\times a\times a\times a}$

$= (a)^{4-4} = (a)^0 = 1.$

Example 5.2

$$(a)^7 \div (a)^7 = \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a \times a \times a \times a}$$

From algebra we know that any number divided by itself is 1 so here we are left with

$$(a)^7 \div (a)^7 = \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a \times a \times a \times a} = 1$$

From rule 2 we also know that $(a)^7 \div (a)^7 = (a)^{7-7} = (a)^0$

Therefore, $(a)^7 \div (a)^7 = \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a \times a \times a \times a} = (a)^{7-7} = (a)^0 = 1$

Can you spot any pattern?

From Examples 5.1 and 5.2 we know that $(a)^0 = 1$.

Therefore any number to the power of 0 is equal to 1.

Rule 5: $(a)^0 = 1$

Example 5.3

Using the rules of indices simplify the following

(i) $(15)^0$

(ii) $(1{,}159)^0$

You can use your calculator to confirm this rule:

(i) Press **15 power button 0 =**

This will give you an answer of 1

(i) Press **1159 power button 0 =**

This will give you an answer of 1

Now you try this.

Fill the empty boxes below by multiplying the two numbers before each empty box:

Here is an example:

(a) $(2)^2$ $(2)^3$

(b) $(3)^7$ $(3)^{10}$

(c) $(6)^4$ $(6)^{12}$

(d) $(4)^7$ $(4)^{13}$

Which is bigger?

i. Three cubed 3^3 or five squared 5^2?

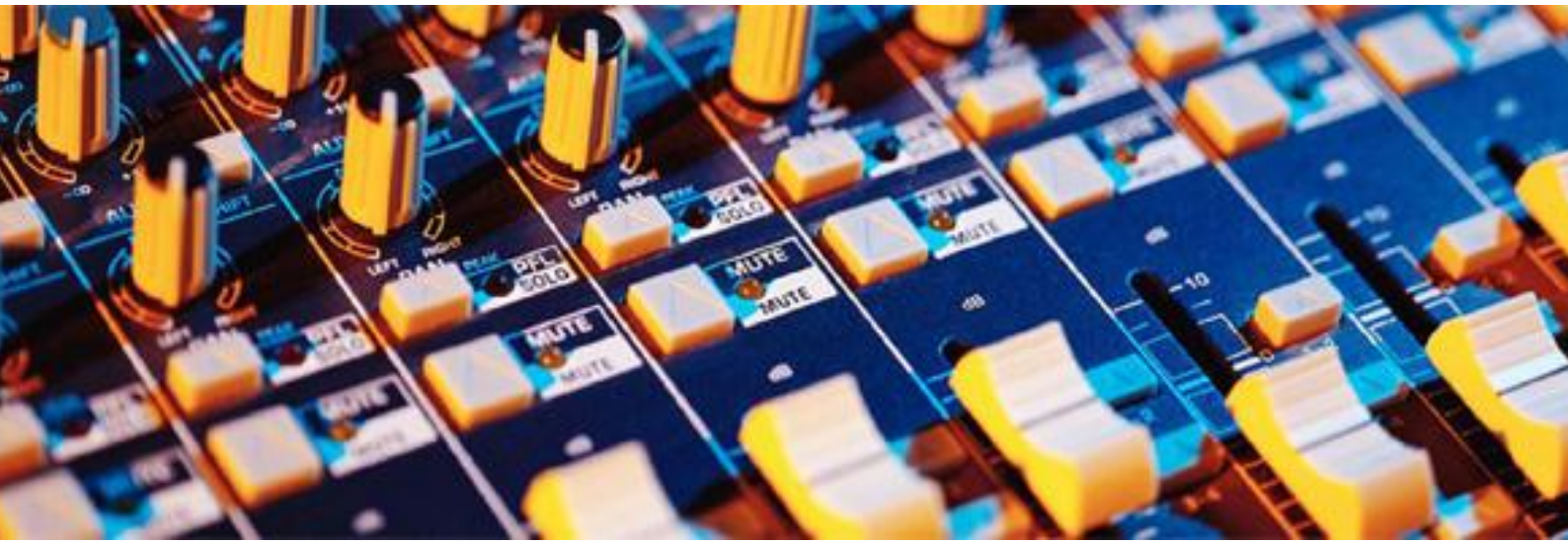
ii. Fifteen squared or six cubed?

Practise your skills

The following practice sheet will help you develop your skills in applying the laws of indices.

- Practice sheet N13

Activity **Concert Volume** **Code N14**



This activity links to award learning outcomes 1.1, 1.4 and 1.5.

Introduction

Writing numbers using index (or power) notation is the shorthand for repeated multiplication. Here we will look at another shorthand for repeated multiplication.
Nowadays we have calculators and computers to help us to do long complicated multiplication but, before these technologies were available people still completed such multiplications. Without calculators or computers, we use our knowledge of **logarithms (logs)** in order to do long, complicated multiplications. Logs were invented by John Napier in the 1600's. Before then, scientists and astronomers spent hours upon hours doing arithmetic. The invention of logs saved a lot of their time and made their work more productive.

Materials you will need

- Calculator
- Pen and paper

Learning Outcomes

1. Understand the laws of logs.
2. Apply the laws of logs.

Key Learning Points

1. Logs

What do you need to know before you start?

Maths

There are 6 **rules of logs** in total. In this section we are going to look at four of these. In order to apply these rules **you need to have completed some of the algebra sections and activity N13**.

Logs are another method of writing indices. There is a connection between logs and indices. In indices the base number is the number that is being multiplied and the power tells how many times we multiply the base number by itself. For example, 10^2: 10 is the base number and 2 is the power.

We can rewrite 10^2 using logs:

$$10^2 = 100$$

$$\log_{10} 100 = 2$$

The base number in an index number is also the base number in a log number.
We say this log number as '*log 100 to the base 10 = 2'.*

Richter Scale

The Richter scale measures the severity of earthquakes.
It was developed by Charles Richter in 1935.
The Richter scale is based on subtracting two logs from each other.

pH Scale

The pH scale measures how acidic or basic a substance is. It is also a log scale. The pH scale is determined as the negative log of hydrogen ion concentration.

Decibel Scale

The decibel scale measures sound levels and is widely used in electronics, signals and communication. Changes in sound pressure affect volume. This is measured by a log ratio.

Task 1: Adding and Subtracting Logs

Rule 1 and Rule 2

Example

The Richter scale measures the magnitude of earthquakes by subtracting one log number from another.

On the 12th of January 2010 an earthquake struck Haiti killing hundreds of thousands of people.

The following measurements were calculated to find out what measure the earthquake in Haiti was on the Richter scale:

$$\text{to } \log_{10}66005 - \log_{10}0.00649$$

(i) What value on the Richter scale did this earthquake measure?

Scientists around the world are interested in predicting earthquakes.

The Richter scale is: **Earthquake = log A – log B**

'A' measures what is called the maximum excursion of the Wood-Anderson seismonograph.

(ii) An earthquake measured $\log_{10} 398107.17$ on the Richter scale and log of B was measured as $\log_{10} 0.0038$

What is log A?

Solution

(i) Earthquake = $\log_{10} 66005 - \log_{10} 0.00649$

$\log_{10}66005 - \log_{10}0.00649 = \log_{10}(66005/0.00649)$ (Rule Number 1)

$= \log_{10}(10{,}170{,}261.94)$

$= 7$

The Haiti earthquake measured 7 on the Richter Scale.

(ii) Earthquake = $\log_{10} A - \log_{10} B$

The earthquake measured $\log_{10}$ 398107.17 on the Richter scale and Log B = $\log_{10}$ 0.0038

$\log_{10}$ 398107.17 = $\log_{10} A$ - $\log_{10}$ 0.0038

Using Algebra we can rearrange this formula so that log A is on its own:

$\log_{10} 398107.17 + \log_{10}0.0038 = \log_{10} A$

$\log_{10} 398107.17(0.0038) = \log_{10} A$

$\log_{10} 1512.8 = \log_{10} A$

From this we can see that A = 1512.8.

To solve for **log A**, we can use a calculator:

The Log button in your calculator is fixed as having a base number of 10. Therefore when working with a base number of ten you can easily solve logs.

In your calculator press **log 1512.8 =**

This will give you an answer of 3.179. Therefore **Log A = 3.18**

Task 2: Rock concert

Rule 3

Example

The volume at a rock concert is not allowed to exceed **120dB** (Decibels).

An acoustic engineer tests the volume before every concert to make sure it is below 120dB. The volume is measured in terms of Sound Pressure.

The volume is calculated as $20\log\left(\frac{pressure_2}{pressure_1}\right)$.

At the concert venue, Hard Rock, the acoustic engineer gives the manager the following equation:

$$\log\left(\frac{740.95}{240}\right)^{20}$$

Is this within acceptable sound levels?

Solution

$$\text{Volume} = \log\left(\frac{673.59}{0.00379}\right)^{20}$$

$$= 20\left(\log\left(\frac{673.59}{0.00379}\right)\right) \qquad \text{(Rule Number 3)}$$

$$= 20 \text{ (log } 177728.23$$

$$= 20 \text{ (5.25)}$$

$$= 105$$

The noise levels created at Hard Rock measure 105dB. So they are within acceptable noise levels and the concert can go ahead.

Task 3

Rule 4

In your calculator check what answer you get for $\log_{10} 10$.

This is one of the rules of logs:

$$\log_a a = 1$$

Therefore, what answer would you expect to get for the following?

(i) $\log_5 5 =$

(ii) $\log_{215} 215 =$

(iii) $\log_{37} 37 =$

The rules of logs

Rule 1 $$\log_a n - \log_a m = \log_a \frac{n}{m}$$

Rule 2 $$\log_a n + \log_a m = \log_a nm$$

Rule 3 $$\log_a b^c = c \log_a b$$

Rule 4 $$\log_a a = 1$$

Now you try this.

pH scale

$$pH = -\log_{10}(\text{hydrogenion cpncentration})$$

What is the pH of lemon juice if the hydrogen ion concentration is 1×10^{-2} ?

_____

Decibel scale

Carbon and Headbash are concerts that are held every year in Ireland.

The sound engineer for Carbon has calculated that the noise levels at Carbon are equal to

$$\log\left(\frac{1451.87}{0.00578}\right)^{20}.$$

The sound engineer for Headbash has calculated that the noise levels are equal to

$$\log\left(\frac{13842.87}{0.098}\right)^{20}.$$

What is the difference in dB levels between the noise at the two concerts?

Which concert is more damaging to the human ear?

Practise your skills

.

- Practice sheet N14

Activity **Earthquake** **Code N15**

This activity links to award learning outcomes 1.1, 1.4 and 1.5.

Introduction

From N13 and N14 you know how to work with logs and indices and you also know that there is a connection between logs and indices. In this section you will learn more about this connection.

Materials you will need

- Calculator
- Pen and Paper

Learning Outcomes

1. Use logs and indices to solve equations.

Key Learning Points

2. Logs
3. Indices

What do you need to know before you start?

Maths

In N14 we saw that we can rewrite an index number as a log number.

For example 5^3: Here 5 is the base number and 3 is the power. We can rewrite this number using logs:

$$5^3 = 125$$

$$\log_5 125 = 3$$

In this example 3 is the power that we must raise 5 to in order to get a value of 125.

When solving problems such as $\log_3 27 = ?$ we ask ourselves this question:

What power do I need to raise the base number (3) by in order to get a value of 27?

That is, $3^?$ = 27.

The power required is 3, therefore $\log_3 27 = 3$.

Task 1: Switching between Logs & Indices

Example 1

Rewrite the following as logs:

(i) 6^2

(ii) 2^3

(iii) 4^3

Solution

(i) 6^2 = 6 x 6 = 36

6^2 = 36 (base number = 6, power = 2)

Therefore as a log: $\log_6 36 = 2$

(ii) 2^3 = 2 x 2 x 2 = 8

2^3 = 8 (base number = 2, power = 3)

Therefore as a log: $\log_2 8 = 3$

(iii) 4^3 = 4 x 4 x 4 = 64

4^3 = 64 (base number = 4, power = 3)

Therefore as a log: $\log_4 64 = 3$

Example 2

Rewrite the following as indices:

(i) $\log_5 625 = 4$

(ii) $\log_2 128 = 7$

(ii) $\log_3 243 = 5$

Solution

(i) $\log_5 625 = 4$ (base number = 5, power = 4)

Therefore as an index: $5^4 = 625$

(ii) $\log_2 128 = 7$ (base number = 2, power = 7)

Therefore as an index: $2^7 = 128$

(iii) $\log_3 243 = 5$ (base number = 3, power = 5)

Therefore as an index: $3^5 = 243$

Task 2: Using Logs & Indices

Example 1

Two small earthquakes occur in the same city within two days of each other. The first earthquake measures $\log_{10} 6.3$ and the second earthquake measures $\log_{10}$ A, where A = $10^{1.1}$

1. **What was the measure of both earthquakes on the Richter Scale?**

Solution

(i) Earthquake 1 = $\log_{10} 6.3$

This log has base number = 10 therefore you can use your calculator. In your calculator press **log 6.3 =**

This will give you an answer of 0.7993

Therefore this earthquake measured 0.8 on the Richter Scale.

Earthquake 2 = $\log_{10}$ A
A = $10^{1.1}$

In your calculator press **10 power button 1.1 =**

$10^{1.1}$ = 12.59

Therefore earthquake 2 = $\log_{10}$ 12.59

In your calculator press **log 12.59 =**

This will give you an answer of 1.1

Therefore this earthquake measured 1.1 on the Richter scale.

Example 2

Solve the following equations:

(i) $3^x = 211$

(ii) $\log_2 x = 5$

Solution

(i) $3^x = 211$

In this question the power is unknown. From algebra we know that we cannot separate the base number and the power easily. When we wish to remove the power or index numbers we can do so by introducing logs.

$\log_{10} 3^x = \log_{10}(211)$

$x \log_{10} 3 = \log_{10} 211$ (Use Log Rule 3).

$x \log_{10} 3 = 2.324$ (Press **log 211** in the calculator).

$x(0.477) = 2.324$ (Press **log 3** in the calculator).

$x = \frac{2.324}{0.477}$

$x = 4.87$

(ii) $\log_2 x = 5$

Rewrite the log number as an index number:

Base number = 2, power = 5

$x = 2^5$

$x = 32$

Practise your skills

- Practice sheet N15

Activity # Interest Free Shopping **Code N16**

This activity links to award learning outcomes 1.1, 1.3, 1.4, 1.6 and 1.7.

Introduction

Whenever you borrow money from the bank it costs money. This cost is called **interest.** It also works the other way around for savings. When you open a savings account you get paid interest. Interest is the money you are charged for borrowing money **or** the money that you are paid for saving money.

Materials you will need

- Calculator
- Pen and Paper

Learning Outcomes

1. Understand the concept of simple interest.
2. Calculate simple interest.

Key Learning Points

1. Interest
2. Repayments

What do you need to know before you start?

Maths

If we wish to open a savings account with a bank we have a number of choices. Different banks will offer different rates and types of **interest**. This interest is added to the savings at the end of the year. We need to find out which bank is offering the most amount of money for our savings.

If we need to borrow money from the bank we must pay interest to the bank. So this time we would need to know which bank is charging the least amount of interest, so that we pay as little extra money as possible.

A sum of money that we borrow or save or invest is called a **principal** amount. We calculate how much interest is earned or paid based on that principal amount. Interest is a percentage of the principal. This percentage is called a **rate of interest**.

Task 1: Calculating Simple Interest

Example 1

Simple interest is calculated once a year on the initial amount that is either invested in a savings account or borrowed.

For example:

You open a savings account and decide to save in it for a period of four years. You lodge **€1,000** into this account, so €1,000 is the **principal** amount. You have found out that the rate of simple interest is 5%. So you are **saving €1,000 @ 5% simple interest.**

At the end of the first year you will earn €50 interest on your principal amount of €1,000.

For the following years you will continue to earn €50 interest on your principal amount of €1,000. Any interest you earned and kept in your savings account will not earn any additional interest.

Therefore, at the end of the four years you will have your principal amount of €1,000 plus the €200 that you have earned in interest, giving you €1,200.

Here is a formula for calculating simple interest:

$$I = \frac{R}{100} \times P \times T$$

I is the amount of simple interest earned. **R** is the rate of interest. **P** is the principal amount. **T** is the number of years.

We can use the formula to calculate the simple interest earned in the example mentioned above.

I = simple interest

R = 5%

P = 1,000

T = 4

$$I = \frac{5}{100} \times 1{,}000 \times 4$$

$$I = \frac{5}{100} \times 4{,}000$$

$$I = 0.05 \times 4{,}000$$

$$I = 200$$

I represents the amount of simple interest. I = 200, which means that your savings earned €200 in simple interest over the four year period.

Credit Cards

- Buying goods or products on your credit card is similar to borrowing money.
- Interest is paid to the credit card company depending on the principal amount (amount owed on your credit card).

Task 1: Calculating Simple Interest

Example 2

How much interest you would have to pay the bank when you borrow €2,500 at 3% simple interest over five years?

Remember:

- The simple interest formula
- What each letter in the formula represents

Solution

I = simple interest
R = 3%
P = 2,500
T = 5

$$I = \frac{3}{100} \times 2{,}500 \times 5$$

$$I = \frac{3}{100} \times 12{,}500$$

$$I = 0.03 \times 12{,}500$$

$$I = 375$$

Task 2: Calculating the amount

When you borrow money the total amount you pay back is called the **amount.** The amount is equal to **the principal plus the interest**. This is also the case when you save money, the **amount** you have at the end of the period is the principal plus the interest.

Example 1

An electrical store has a sale offer on plasma TV's. The TV's can be bought for €900 cash **or** 12 monthly payments of €89. **Calculate how much interest is charged** if you choose to buy the TV using the 12 monthly repayments?

Solution

Amount = Principal + Interest

Total amount paid is €89 x 12 (repayments) = €1,068
Principal is the initial amount (cost of the TV) = €900

Amount = Principal + Interest
€1,068 = €900 + I
€1,068 - €900 = €900 - €900 + I
€168 = I

Therefore the total interest paid would be €168.

Task 2: Calculating the amount

Example 2

Cost Minus Sofas is a furniture shop. It normally charges **12.5% interest** for credit. This shop is currently advertising a one year interest free repayment plan. So, you decide to buy a new three piece suite of furniture. The cost of the suite is **€1,440.** You will pay for the furniture using **12 monthly repayments**. This will mean that you will have paid for the sofa by the end of one year and will not have to pay interest.

i. How much will each repayment need to be?
ii. How much have you saved on interest by buying while the shop was offering interest free shopping for 12 months?

Calculate the repayment and then calculate the simple interest.

_____

Solution

(i) You must pay back €1,450 in 12 equal repayments.
€1,440 ÷ 12 = €120

(ii) I = simple interest
R = 12.5%
P = 1,440
T = 1

$$I = \frac{12.5}{100} \times 1{,}440 \times 1$$

$$I = 0.125 \times 1{,}440$$

$$I = 180$$

Therefore you have saved €180 by buying while the one year interest free shopping was offered.

Now you try this

A washing machine costing €650 can be bought in 24 monthly repayments of €32.50. How much interest does the shop charge for purchasing the washing machine using the 24 repayments?

Seán is buying a new car. It costs €12,000 and the garage will give him €9,000 for his old car. Seán needs to borrow the remaining amount from the bank.

How much does Seán need to borrow?

Seán takes out this loan for 5 years with simple interest and he pays the bank back a total of €3,600.

How much interest did Seán pay back?

What was the rate of simple interest that the bank charged Sean?

Practise your skills

Practice sheet N16

Activity **Choosing Bank Products** **Code N17**

This activity links to award learning outcomes 1.1, 1.3, 1.4, 1.6 and 1.7.

Introduction

Compound interest differs from simple interest. When you save money at compound interest rates you can earn interest on your interest. **The interest earned at the end of each year is added to the principal amount and this is reinvested as a new lump sum the next year.**

Materials you will need

- Calculator
- Pen and Paper

Learning Outcomes

1. Understand the concept of compound interest.
2. Calculate compound interest.

Key Learning Points

1. Compound interest
2. Repayments

What do you need to know before you start?

Maths

Compound interest is calculated once a year on the principal plus any interest which may have already been earned.

Like simple interest, compound interest must be paid on money that you borrow and is earned on money that you save. The interest rates quoted on investments, credit cards, loans, store credit, overdrafts etc are always compound interest unless otherwise stated. When we hear people on TV or the radio talking about interest rates they are talking about **compound interest rates.**

Compound interest can be calculated annually or more often.
Annually means once a year.

Calculating compound interest annually

Suppose you want to invest **€6,000 for 3 years at 4.5% p.a. (per annum)**. At the end of the first year your €6,000 will earn €270 interest. This interest is added to your principal and reinvested for year two. Therefore in year two the interest earned will be based on your new principal of €6,270.

Year	Principal at the start of the year	Interest earned	Amount at the end of the year
1	€6,000	€270	€6,270
2	€6,270	€282.15	€6,552.15
3	€6,552.15	€294.85	€6,847

Note: We use the method of simple interest to calculate the interest earned each year.

Calculating compound interest more often than annually

In reality banks calculate compound interest more than once a year. It is normally calculated **monthly** (12 times a year which is once a month), **quarterly** (four times a year which is every three months) or **bi-annually** (twice in a year which is every six months).

For example: Suppose you want to invest **€2,000 for 1½ years at 4% p.a. compound interest** which is compounded **bi-annually**. Interest is applied every six months. So that means interest is applied 3 times in the 1½ year period.

Period	**Principal at the start of the period**	**Interest** $I = \frac{R}{100} \times P \times T$	**Interest earned**	**Amount at the end of the period**
1	€2,000	$I = \frac{4}{100} \times 2{,}000 \times 0.5$	€40	€2,040
2	€2,040	$I = \frac{4}{100} \times 2{,}040 \times 0.5$	€40.80	€2,080.80
3	€2,080.80	$I = \frac{4}{100} \times 2{,}040.80 \times 0.5$	€41.62	€2,122.42

T represents time in years and the time in years for each of these periods is half a year (0.5)

Here is a formula for calculating compound interest which is compounded annually **or** more frequently:

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

A is the total **amount** repaid or the total savings including interest. **R** is the rate of interest. **P** is the principal amount and **n** is the number of years (or other periods of time such as months).

We can use this formula to calculate the compound interest earned.

A = amount

R = 5%

P = 1,000

n = 4

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$A = 1{,}000\left(1+\frac{5}{100}\right)^{4}$$

$$A = 1{,}000\,(1+0.05)^{4}$$
$$A = 1{,}000\,(1.05)^{4}$$
$$A = 1{,}000\,(1.216)$$
$$A = 1{,}216$$

If A = Principal + Interest, then Interest = Amount – Principal.

Therefore the total interest paid is €1,216 - €1,000 = €**216**

If we wish to calculate interest more often than once a year – for example, monthly, quarterly or bi-annually - then the **p.a.** interest rate is affected.

For example: A bank is offering 4.2% p.a. compound interest compounded monthly. If you deposit €1,000 for two years, how much interest would you earn?

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

A = Amount

P = 1,000

R = 4.2% p.a.

Remember: **p.a. means** per annum, or **once a year.**

But it is now being applied **monthly,** which is **12 times a year.**
Therefore the new R value is **4.2%÷12** which is **0.35% per month**.

In this example, n = 12 periods per year.

So for two years n = 12 x 2 = **24**

$$A = 1{,}000\left(1+\frac{0.35}{100}\right)^{24}$$

$$A = 1{,}000\,(1+0.0035)^{24}$$

$$A = 1{,}000\,(1.0035)^{24}$$

$$A = 1{,}000\,(1.088)$$
$$A = 1{,}088$$

Amount = Principal + Interest

Interest = Amount – Principal
Interest = €1,088 - €1,000 **= €88**

Task 1: Calculating Compound Interest

Example

A Building Society pays 4.2% p.a. compound interest on deposits over €4,000. If you deposit €5,800 for three years,

a. How much interest would you earn if the interest is calculated annually?

b. How much interest would you earn if the interest in calculated quarterly?

   Quarterly means four times a year.

Remember:

- You can calculate (a) on a year by year basis or using the compound interest formula.
- For part (b) remember that a per annum rate is affected when the interest is applied more than once a year.

Task 1: Calculating Compound Interest

Solutions

Solution (a)

A = Amount R = 4.2% P = 5,800 T = 3

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$A = 5{,}800\left(1+\frac{4.2}{100}\right)^{3}$$

$$A = 5{,}800\left(1+0.042\right)^{3}$$
$$A = 5{,}800\left(1.042\right)^{3}$$
$$A = 5{,}800\left(1.131\right)$$
$$A = 6{,}559.8$$

Amount = Principal + Interest

Therefore:

Interest = Amount – Principal

Interest = €6,559.80 - €5,800 = €759.80

Solution (b)

A = Amount R = 4.2% ÷ 4 = 1.05%

P = 5,800 n = 4 times per year, for 3 years n = 4 x 3 = 12

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$A = 5{,}800\left(1+\frac{1.05}{100}\right)^{12}$$

$$A = 5{,}800\left(1+0.0105\right)^{12}$$
$$A = 5{,}800\left(1.0105\right)^{12}$$
$$A = 5{,}800\left(1.134\right)$$
$$A = 6{,}577.20$$

Amount = Principal + Interest

Therefore:

Interest = Amount – Principal

Interest = €6,577.20 - €5,800 = €777.20

Task 2

Choosing between Simple and Compound Interest

Example 1

ABI Bank offers 5% simple interest p.a. on any savings accounts greater than €2,000 and less than €10,000. BEI Bank offers 4.8% compounded interest p.a., compounded bi-annually, on any savings accounts greater than €2,000 and less than €10,000. You wish to deposit €6,000 for four years. Which bank has the best offer?

Calculate the interest for both banks and then compare.

_____

Choosing between Simple and Compound Interest

Solution Example 1

ABI Bank

Simple Interest: $I = \frac{R}{100} \times P \times T$

$$I = \frac{5}{100} \times 6{,}000 \times 4$$

$$I = 0.05 \times 24{,}000$$

$$I = 1{,}200$$

BEI Bank

Compound Interest

A = amount R = 4.8% p.a. ÷ 2 = 2.4% bi-annually

P = 6,000 n = twice each year, 4 x 2 = 8 periods

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$A = 6{,}000\left(1 + \frac{2.4}{100}\right)^8$$

$$A = 6{,}000\left(1 + 0.024\right)^8$$

$$A = 6{,}000(1.024)^8$$

$$A = 6{,}000(1.209)$$

$$A = 7{,}254$$

Interest = Amount – Principal

Interest = €7,254 - €6,000 = €1,254

ABI Bank is offering €1,200 in interest on €6,000 for four years but BEI Bank is offering €1,254 for the same amount and time. Therefore BEI Bank is giving the better offer.

Choosing between Simple and Compound Interest

Example 2

Kate has just turned 38 and wants to save for the next two years so that she can go on a holiday to Florida for her 40th birthday. The total cost of the trip will be €6,000. Kate has a lump sum saved already but it is not enough so she has decided to invest this lump sum into a savings account with ABI Bank. ABI bank has offered Kate a **3% p.a. compound interest rate**, and the interest is compounded **quarterly**. This interest rate over the time period will ensure that her **principal** lump sum will **amount** to €6,000. **How much money did Kate invest** in her savings account? **Calculate the principal value (P).**

Choosing between Simple and Compound Interest

Solution Example 2

Compound Interest

A = 6,000 R = 3% p.a. ÷ 4 = 0.75% quarterly

P = principal n = four times a year, 2 years x 4 = 8 periods

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$6{,}000 = P\left(1+\frac{0.75}{100}\right)^{8}$$

$$6{,}000 = P\left(1+0.0075\right)^{8}$$

$$6{,}000 = P\left(1.0075\right)^{8}$$

$$6{,}000 = P(1.0616)$$

$$\frac{6{,}000}{1.0616} = \frac{P(1.0616)}{1.0616}$$

$$5{,}651.85 = P$$

Therefore Kate had €5,651.85.

Now you try this.

Tim has just recently opened his own shop. He purchased various shop fittings from Shops R US for his shop which cost him €6,000. Shops R US gave him two years interest free credit (i.e. he does not have to repay the €6,000 for two years).

Tim is going to lodge a lump sum with BEI Bank in order to meet this payment in two years. BEI Bank are offering Tim 4% p.a. compound interest compounded quarterly.

How much does Tim need to lodge now in order to have €6,000 in two years?

_____

A credit card company charges 3.5% p.a. compounded interest which is compounded monthly on the balance owed each month. Lucy decided to treat herself this month and went on a shopping spree. She spent €215.90 on her credit card. How much interest will Lucy have to pay if she doesn't clear her credit card before the end of the month?

_____

Practise your skills

- Practice sheet N17

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