Resources Pack Level 4 Mathematics





Level 4: Mathematics Unit 1: Number

Use your calculator to complete the following:

(a) 1.45 + 0.89 x 4=	(b) $\frac{2}{7} \times \frac{1}{7} - 0.00035 =$	
(c) $2 - 3 \times 5 + 6 =$	(d) 19.875 ÷ 4 + (2 x 1)=	
(e) $(\sqrt{289} - 3) \div 2 =$	(f) 25% of 165=	
(g) $\frac{4}{11}$ of €4.79 =	(h) $4(\pi)(3^2) =$	
(i) $\frac{1}{3}x\frac{2}{3} + \frac{1}{3} =$	(j) $\frac{2}{5} \times 1.75 - 0.75 =$	
(k) 279÷ π =	(I) $\frac{4}{11} \div 1.64 - 3 \times 0 =$	
(m) 13% of 169 =	(n) $\sqrt{3^2 + 16} =$	

Question 1

Evaluate the following:		
(a) 14 x 12	(b) 23 x 16	
(c) 44 x 18	(d) 16 x 14	
(e) 21 x 11	(f) 32 x 34	
(g) 18 x 45	(h) 7 x 32	

Question 2

Evaluate the following:

(a) 256 ÷ 16	(b) 180 ÷ 18	
(c) 75 ÷ 25	(d) 272 ÷ 17	
(e) 450 ÷ 25	(f) 1152 ÷ 36	
(g) 4500 ÷ 90	(h) 1320÷ 12	

Question 3



Question 1

Convert the following percentages to fractions in their simplest form:

(a) 75%	(b) 18%	
(c) 22%	(d) 35%	
(e) 20%	(f) 19%	
(g) 85%	(h) 40%	
(i) 98%	(j) 48%	
(k) 16%	() 11%	
(m) 33%	(n) 90%	

Question 2

Evaluate the following:

(a) 20% of 100	
(b) 30% of 50	
(c) 5% of 200	
(d) 14% of 500	
(e) 90% of 200	
(f) 17% of 150	
(g) 26% of 560	
(h) 72% of 340	
(i) 60% of 900	

Question 1

Write the following as fractions in their simplest form:



- (c) 6 out of 12
- (e) 12 out of 20



(i) 22 out of 66





(j) 13 out of 67



Question 2

Rewrite the following fractions as percentages:



Question 1

Convert the following decimals to fractions in their simplest form:

(a) 0.4	(b) 0.18	
(c) 0.65	(d) 0.125	
(e) 0.878	(f) 0.9	
(g) 0.32	(h) 0.97	
(i) 0.256	(j) 0.25	
(k) 0.216	(1) 0.85	
(m) 0.36	(n) 0.001	

Question 2

Convert the following decimals to percentages:

(a) 0.65	(b) 0.3	
(c) 0.58	(d) 0.365	
(e) 0.423	(f) 0.08	
(g) 0.63	(h) 0.158	
(i) 0.001	(j) 0.09	
(k) 0.26	(I) 0.525	
(m) 0.082	(n) 1.0	

Calculate the gross income for the following people:

- Nancy is currently working as a substitute tutor. She gets paid by the hour. The government currently pays substitute tutors €28.50 per hour. Nancy worked 7.25 hours last week. What was her gross pay?
- John works with his father as an electrician. He works 37 hours a week and gets paid €9.10 an hour. Last week he had to work 4 hours overtime on Sunday and he gets paid time and half for overtime. What was John's gross pay last week?

Calculate the total deductions for the following people:

- Keith has a gross pay of €367 and a tax credit of €75. He pays 20% Income Levy and 20% PAYE. What are Keith's total deductions?
- Maura has a gross pay of €515 and a tax credit of €104. She pays 2% Income Levy, 21% PAYE and 5.8% PRSI. She also contributes €1.90 to her Trade Union. What are Maura's total deductions?

Calculate the following:

- Tim works in a local factory. He has a net pay of €279.50. His total deductions were €98.75.
 What is his gross pay?
- Maeve is an auctioneer; she makes 1.7% commission on each house she sells. She sold a house this week for €155,000. What was her commission?
- Jerry is a sales representative for a farm machine company. His monthly salary is €1,200. He earns 4.9% commission on any machinery he sells. In August he sold €24,580 worth of machinery. What was his gross salary in August? He pays 2% Income Levy, 20% PAYE and 6% PRSI. If his tax credit is €321, what was his net pay in August?

- A strawberry picker earns 21 cent per punnet of strawberries picked. On a good day a strawberry picker picked 213 punnets of strawberries. How much did he earn?
- Thomas is an auctioneer. He charges 1.75% commission on the sale price of a house.
 Calculate Thomas's commission on the sale of a house for €115,850.
- Una runs a stationery shop in her local town. Her total sales for 2010 were €58,850. The cost of her goods was €23,987
 - Calculate Una's gross profit
 - o Una's total operating costs came to €16,107 in 2010. What was Una's salary?
- Catherine is retiling her kitchen. She has hired Peter to do this job. He gave her the following estimate for the job:

9 hours at €9.50 an hour
6 boxes of tiles @ €42.15 per box
3 tubs of tile adhesive @ €6.20 per tube
Fitting: 12m² this costs €12.49
VAT @ 13.5%

- When the job is finished there is one box of tiles left over and 0.75 of a tube of adhesive left over.
 - Calculate the total cost of the job
 - Calculate the total cost of wastage
- Joe is a dairy framer. He received €97,650 in payments in 2009. His operating expenses are as follows:

Equipment	€24,560	Light and heat	€1,243
Transport	€4,300	Fertiliser	€3,250
Feeding	€3,940	Veterinary Bills	€2,875

- Draw up Joe's profit and loss account
- What is Joe's gross salary if he reinvests 12% of his profits?

Question 1

Simplify the following ratios (if possible):

(a) 14:7	(b) 16:4	
(c) 11:22	(d) 18:6	
(e) 2:1	(f) 15:5	
(g) 4:12	(h) 24:6	
(i) 3:18	(j) 12:48	

Question 2

Write the following ratios in fraction form:



Question 1

The following table represents Euro exchange rates:

Currency	We Sell	We Buy
Sterling	0.835	0.8586
US Dollars	1.3406	1.3786
Australian Dollars	1.3633	1.4019
Thai Baht	40.0648	41.2024
Swiss Franc	1.3202	1.3576
Norwegian Krone	8.033	8.261
Indian Rupee	60.7154	62.4394

Source: <u>www.aib.ie</u> (16th November 2010)

Using the table above convert the following:

- (a) €25 to Australian Dollars
- (b) €100 to Indian Rupee
- (c) €10 to Thai Baht
- (d) £50 Sterling to Euro
- (e) 40 US Dollars to Euro
- (f) €36 to Norwegian Krone
- (g) €57 to Swiss Franc
- (h) £10 Sterling to US Dollars
- (i) 20 American Dollars to Indian Rupee
- (j) 50 Australian Dollars to Thai Baht
- (k) 15 Norwegian Krone to Swiss Franc

* Hint: For the last 4 questions convert the money to euro first, round your answers to two decimal places and then convert from euro to the required currency.



Question 1

Change the following numbers into scientific notation:

(a) 1000	(b) 3890	
(c) 5,000,000	(d) 4,500,000	
(e) 670,000	(f) 56,789	
(g) 5,342	(h) 69,534,000	
(i) 78,901	(j) 4,670,000	
(k) 0.00056	(I) 0.00001	
(m) 0.23	(n) 0.00605	

Question 2

Change the following numbers from scientific notation into standard form:



Question 1

Round off the following to **<u>one</u>** decimal place:

(a) 3.66	(b) 4.91	
(c) 8.756	(d) 13.487	
(e) 11.608	(f) 16.182	
(g) 23.478	(h) 36.828	
(i) 71.419	(j) 8.673	

Question 2

Round off the following to **two** decimal places:

(a) 1.653	(b) 8.597	
(c) 15.6395	(d) 11.5937	
(e) 27.21593	(f) 95.6389	
(g) 5.6926	(h) 32.6512	
(i) 26.95341	(j) 11.52392	

Question 3

Round off the following to **three** decimal places:

(a) 18.65239

(c) 13.958633

(e) 19.53926

(g) 11.23897



(b) 1.23698	
(d) 52.63231	
(f) 56.36825	
(h) 12.3659	

Question 1

Calculate the percentage error between the estimate and the actual value of the different items below:

Item	Estimate	Actual Value	Percentage Error
Box of Cereal	€2.50	€3.20	
Adidas Runners	€40	€55	
HD Television	€500	€480	
GAA All Ireland Final	€56	€70	
Ticket			
16GB Ipod Nano	€200	€170	
Nintendo Wii Console	€185	€200	
XFactor Live Tour	€45	€54	
Tickets			
Pre-owned Nissan	€3,500	€3,750	
Micra (2003 model)			

Level 4	Re	sources	Mathematics	
Practice Sheet N13				
Without using your ca	lculator simplify the	e following:		
(a) $3^2 \times 3^7 =$		(b) $4^3.4^{11} =$		
(c) $3^7 \div 3^2 =$		(d) $(3^7)^3 =$		
$(e)^{(3x)^3} =$		(f) $5^0 =$		
(g) $7^4 \times (7^3.7^2) =$		(h) $(12a)^4 =$		
(i) $(x^a)^3 =$		(j) $7^4 \div (7^3.7^2) =$		
(k) $13^4 \div (13^1.13^3) =$		(1) $8^6 \times 8^{-2} =$		
(m) $8^6 \div 8^{-2} =$		(n) $13^{10} \div (13^1 \div 13^3) =$		

Without using your calculator simplify the following:



Level 4	Resources	Mathematics		
Practice Sheet N15				
Express the following as	indices:			
$(a)^{\log_{17} 83521} = 4$	(b) $\log_{10} 560 =$			
(c) $\log_5 0.008 = -3$	(d) $\log_{0.5} 0.25 = 2$			
Express the following as	s logs:			
(e) $4^3 = 64$	(f) $13^4 = 28,561$			
(g) ^{3⁵=}	(h) $154^{0.5} =$			
Use Logs and Indices to solve the following:				
$(i)^{2^x} = 64$	(j) $\log_3 729 = x$			
(k) $3^x = 1.732$	(1) $\log_6 4.193 = x$			
(m) $\log_{10} 8^x = \log_{10} 3.784$	(n) $\log_{10} 156 = x$			

Calculate the simple interest which is paid on the following loans:



Calculate the simple interest which is earned on the following savings:



Calculate the amount (A):



Calculate the compound interest p.a. which is paid on the following loans:



Calculate the compound interest which is earned on the following savings:

(d) P = €5,890 R = 3% T = one year	
Compounded = me	onthly
(e) P = €6,250 R = 3.6% T = two years Compounded = bi-	annually
(f) P = €5,300 R = 3.5% T = Two years	

Compounded = quarterly

(g) €890 is invested for 4 years at 4.5% p.a. compounded interest. Interest is compounded biannually. What is the amount at the end of the time period? (h) What principal amount (P) will become €6,000 in two years when it is invested at 3%p.a. compounded interest? Interest is compounded quarterly.

(i) Which costs the borrower more: a simple interest loan of \in 3,000 for two years at 3.1% simple interest or a compounded interest loan of \in 3,000 over two years at 2.9% p.a. compound interest which is compounded monthly?



Level 4: Mathematics Unit 2: Geometry

In the space on the right name each of the following shapes.



(g)



(h)



















State if each figure below has folding symmetry. If it has, then indicate the line that gives this symmetry.





(b) Plot the points (-2, 3), (-1, 4), (3, -1)







(e) A is the point (1, 3). B is the point (3, 1). Label these points in the diagram below:



(a) A group of people get together to support their local hospital. They plan a walk and look for sponsorship based on a rate of €2.50 for every mile walked. The following table shows the amounts of money collected and the miles walked by a number of people in the group.

Amount of money in €	2.5	5	7.5	10	12.5
Miles walked	1	2	3	4	5

Draw a graph of the results with the Miles walked on the *x*-axis.

(b) A new born baby weighs 7 pounds. Over the course of the first ten weeks of her life she gains $\frac{2}{3}$ of a pound each week. The graph below shows the baby's weight plotted against her age.



What weight is the baby after three weeks?

How long does it take the baby to reach a weight of eleven pounds?

• Calculate the length of the hypotenuse of a right triangle whose sides are 4 and 7 centimetres respectively.

• A right angled triangle has a hypotenuse of length 5.6 cm. If one side is 4 cm in length how long is the other side?

• A rectangle has sides 5.34cm and 2.78cm. How long is the diagonal?

• Calculate the distance between the points (3, -4) and (-2, 5).

The centre of a circle is at the point (7, -10). It passes through the point (1, 3). What is the radius of the circle?

• A rectangle has vertices at the points shown in the diagram. How long is the diagonal AC?



• Find the **centre** of the line segment joining the points (2, 6) and (8, 10).

The diameter of a circle has one end at the point (-7,8) while the other end touches the point (3,-6). What are the coordinates of the centre of the circle?

• A square is formed by joining the points (0, 0), (0, 4), (4, 4) and (4, 0). Where do the diagonals of the square meet?

- Find the **slope** of the line segment joining the points (2, 6) and (8, 10).
- The diameter of a circle has one end at the point (-7,8) while the other end touches the point (3,-6). What is the slope of the diameter?

• Find the slope of the line segment joining the points $\left(\frac{3}{2},1\right)$ and $\left(-3,\frac{3}{5}\right)$.

• A line has a slope of 3. One point on the line is (4, 8). A second point on the line has an *x*-coordinate of 1. What is its *y*-coordinate?

• Find the slope of the diameter of a circle going from $\left(\frac{5}{6}, \frac{4}{3}\right)$ to $\left(-\frac{2}{3}, \frac{4}{7}\right)$

• The line segment AB joins the point A (1, -3) to the point B (-2, 5). The line segment CD joins the point C (3, 2) to the point D (-7, -9). Is AB parallel to CD? Explain your answer.

• Do the points (-1, -4), (2, 3), (4, 6), (1, -1) form a parallelogram?

• Do the points A (1,1), B (-1,-3), C $\left(-\frac{1}{2},1\right)$ and D $\left(\frac{1}{2},3\right)$ form a parallelogram?

• The line segment AB joins the point A (1, -3) to the point B (-2, 4). Find the slope of the line going through the points A and B. What is the slope of the line perpendicular to AB?

Line segments are drawn joining the points A(1, 1), B(2, 3), C(6, 1), D(5, -1). By calculating the slopes of AB, CD, BC and AD show that the figure is a parallelogram. Use these slopes to show that the parallelogram is a rectangle.

• The point A has coordinates (2, 6). The point B has coordinates (-4, 3). What is the slope of the perpendicular bisector of the segment AB?

The line segment AB joins the point A (1, -3) to the point B (-2, 8). Find the equation of the line AB.

Find the equation of the perpendicular bisector of the line segment PQ where P is the point (-8, 7) and Q is the point (6, -3).

• Where does the line 2x - 3y = 8 meet the x-axis?

• Find the point of intersection of the lines 3x + 4y = 18 and 2x - 5y = -11

- A circle has its centre at the point (0, 0). The radius of the circle is 13. What is the equation of the circle?
- The equation of a circle is $x^2 + y^2 = 225$. What is the radius of the circle?

- What is the equation of a circle with centre at (0, 0) and which passes through the point (3, –
 4)?
- Find the length of the radius of the circle whose equation is $16x^2 + 16y^2 = 289$.
- A circle has its centre at the origin. It passes through the point of intersection of the lines x+3y=-1 and 2x+y=1. Find the equation of the circle.

• A circle $x^2 + y^2 = 25$ contains the point P whose coordinates are (3, -4). What is the equation of the tangent to the circle at the point P?

• The circle $x^2 + y^2 = 1$ contains the point $\left(\frac{3}{5}, -\frac{4}{5}\right)$. Find the equation of the tangent to the circle at this point.

• The line 5x+12y=169 of slope $-\frac{5}{12}$ is a tangent to the circle with centre (0, 0) and radius 13. Where does this tangent meet the circle? Find the equation of the radius at this point.
• Construct a triangle, *ABC*, with |AB| = 5cm, |AC| = 3cm and |BC| = 4cm.

• Construct a triangle ABC having |AB| = 6 cm, $|\angle BAC = 43^{\circ}|$ and |BC| = 8 cm.

• A triangle ABC has side |AC| = 4cm. $|\angle CAB| = 52^{\circ}$ and $|\angle ACB| = 75^{\circ}$. Construct the triangle.

• Construct a triangle ABC where |BC| = 5cm, $|\angle ABC| = 50^{\circ}$ and $|\angle BAC| = 90^{\circ}$.

• The rectangle ABCD has |AB| = 5cm and |BC| = 3cm. Construct ABCD.

• Calculate the area of a rectangle whose sides are 4cm and 7cm.

• A running track is in the shape of a rectangle of sides 100m long by 50m wide. Each end of the track is in the form of a semi-circle as shown in the diagram.



What is the total length of the track?

• A rectangular sheet of metal of sides 1m and 0.5m is melted down. All the metal is used to make a disc. What is the radius of the disc?

• A kitchen wall is divided into two triangles. One triangle contains storage cupboards. The other is plain and needs to be painted. The triangle requiring paint has a horizontal side of length 2.2m and a vertical side of length 3.4m. What is the area that requires paint?

 During the winter a field floods making much of it impossible to use. The field is in the shape of a rectangle with sides 76m and 54m.



The first thing to do next Spring is to divide the field in two by erecting a fence along the diagonal as shown. How long will the fence be?

You want to prepare a quote for the job of painting a house. The first thing you need to do is find out the heights of the walls to help you prepare the quote. The rear of the house backs onto rough ground. You have no way of getting in to measure the wall unless you actually get the contract for the job. You measure the angle that the top of the house makes with the ground from a point *B*. You find that it is 35°. You walk 5.3m closer to the house, which is as close as you can get, and again measure the angle to the top of the house. This time the angle is 68°. Draw a diagram using 1cm for 1m to represent these measurements. Measure the height of the wall from your diagram. How high is the wall of the house?

 A company has an order to manufacture 10000 cans. These cans are cylinders of height 12cm and diameter 8cm.



When you cut the can you need three pieces of metal. The top and bottom of the can are circles with diameter of 8cm. The body of the can is a rectangle. The length of the rectangle is 12cm. The breadth of the rectangle is the circumference of the circle of the top.

What is the breadth of the rectangle?

What is the area of tin needed for one can?

How much metal, in square metres, is needed for the 10000 cans?



Level 4: Mathematics Unit 3: Algebra

Question 1

To play table tennis in the local club, you pay an admission charge of \in 4 and an additional charge of \in 1.50 per hour. The total cost (C) of playing table tennis when t is time in hours, can be expressed by C = 1.5t + 4. Use this expression to calculate the cost of playing table tennis for 3 hours.

Question 2

The cost of a badminton racket is \in 16 and the cost of a shuttlecock is \in 2. The cost (C) of x rackets and y shuttles can be expressed by C = 16x + 2y. Determine the cost when a person buys 6 rackets and 8 shuttles.

Question 3

In soccer, teams are awarded 3 points for games won (w) and one point for games drawn (d). The total points (P) gained by one team can be expressed by P = 3w + d. Use this expression to calculate the total points for Liverpool in the 2008 season in which they won 25 games and drew 11.

Question 4

If a = -1 and b = 2 and c = 4 evaluate the following:

- i. 4c 3b =
- ii. 3b + 2c b =
- iii. -2(a) b =
- iv. a+b+c=
- v. -a-b-c =

1)	x + 7 = 16	
2)	x + 10 = 15	
3)	5 + x = 12	
4)	x - 4 = 8	
5)	x - 2 = 9	
6)	x + 7 = 8	
7)	x - 1 = 14	
8)	x + 565 = 720	
9)	-432 + x = 123	

10) x - 226 = 624

1)	x + 3x = 16	
2)	2x + 10 = 18	
3)	5x + x = 12	
4)	2x - 4 = 8	
5)	3x - 14 = 7	
6)	5x - 7 = 8	
7)	4x - 14 = 2x	
8)	20 = x + x	
9)	45 - 2x = 3x	
10)	-2x + 36 = 4x	

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1)	$\begin{aligned} x + y &= 9\\ x - y &= 3 \end{aligned}$	
2)	2x + y = 8 $3x - y = 2$	
3)	x + 2y = 7 $2x + y = 8$	
4)	4x - y = -9 $2x - 3y - 7$	
5)	2x + y = -2 $x + 3y = 9$	
6)	3x + 4y = 5 $2x - 3y = 9$	
7)	3x + 5y = 6 $2x + 3y = 5$	
8)	2x + 3y = 3 $x - 4y = 7$	
9)	3x - 2y = 13 $4x + 3y = 6$	

10)	2x - 5y = 3
10)	x = 3y + 1

1)	$x + 2 \le 5$	
2)	$x-1 \le 4$	
3)	$2x - 3 \le 5$	
4)	$4x - 5 \ge 3$	
5)	$2x+5 \le 1$	
6)	$3x+4 \ge -5$	
7)	$3x - 5 \le 7$	
8)	$4x + 3 \ge -5$	
9)	$6-x \ge 4$	
10)	$3-x \le 4$	

• When 14 issubtracted from President Mary McAleese's age, the result is 44. How old is the President?

• When 13 is added to Ireland's famous cyclist Sean Kelly's age (x), the result is 66. How old is Sean?

• When 18 is added to the number of red stripes on the American flag the result is 31. How many red stripes are on the flag?

• When 16 is subtracted from Venus Williams age (x), the result is 13. How old is Venus?

• When 12 is added to Sonia O'Sullivan's age (x), the result is 52. How old is Sonia?

• When 13 is added to Jedward's age (x), the result is 31. How old are the twins?







- Dan Shanahan is 11 years older than Joe Canning. If the sum of their ages is 53, find the age of each.
- Grainne Seoige is 4 years older than her sister Sile. If the sum of their ages is 66, find the age of each.
- Roy Keane is 7 years older than Brian O'Driscoll. If the sum of their ages is 69, find the age
 of each.
- Pat Kenny is 25 years older than his successor on The Late Late Show, Ryan Rubridy. If the sum of their ages is 97, find the age of each.
- Padraig Harrington is 18 years older than Rory McElroy. If the sum of their ages is 60, find the age of each.
- Henry Shefflin has 4 All Ireland Hurling medals more than Sean Og O hAilpin. Together they have 10. How many has each?

• The length of a soccer pitch is 35m more than its width and its perimeter is 330m. Find the width of a pitch.

• The length of a rugby pitch is 30m more than its width and its perimeter is 340m. Find the width of a pitch.

• The length of a hockey pitch is 35m more than its width and its perimeter is 290m. Find the width of a pitch.



Level 4: Mathematics Unit 4: Data Handling

Identify the (i) population (ii) the sample (iii) the variable measured and (iv) the statistic provided in the following examples.

• A protester in interested in how many people in her local community are interested in travelling with her to Dublin for protest. She sends out letters to 300 households and receives 200 replies. 75 out of the 200 people who replied would be interested in making the journey to Dublin to protest.

Population:
Sample:
Variable Measured:

Statistic:_____

• A coffee franchise wish to open on a university campus. An email is sent to all university students and staff to ask them if they would buy coffee from this new shop should it open on campus. 3,400 students replied to the email. 3,000 of those that replied said they would buy their coffee from the shop.

Population:_____

Sample:_____

Variable Measured: _____

Statistic:_____

• A primary school class want to measure the growth rate of their plants in school. They decide to put half the plants under sunlight for four hours and the other half under sunlight for 8 hours. They will them compare growth levels.

Can you identify the independent variable and the dependent variable in this study?

Independent variable:

Dependent variable:

 Two classes of students were given a short training course on speech and language. The speech and language tutor wanted to know if an incentive in the form of money would influence the students' performance in the test taken at the end of the course. The students in one class were offered €5 if they got a certain amount of marks on the test while the other class was not offered any money.

Can you identify the independent variable and the dependent variable in this study?

Independent variable:

Dependent variable:

• A number of parents were asked how long their children play video games for per week. These were the results (in hours):

1	2	2.5
3	3.75	3
1.5	2	4.5
4	3.20	4
1	0.5	2

How many parents were in the survey?

Is the data discrete or continuous? Why?

• The speed cameras installed on a motorway recorded the speed, to the nearest kilometre, that a number of cars were travelling at over the course of an hour. These were the results (in kilometres):

110	85	98
120	123	109
89	96	113
75	115	105

How many cars were in the survey?

Is the data discrete or continuous? Why?

• A number of restaurants were asked what their most popular dish was. The results were as follows:

Complete the following frequency table:

Favourite	Lasagne	Vol au	Steak	Chicken	Beef	Vegetarian
Dishes		vents		Curry	Burger	Stir Fry
Number of						
Restaurants						

How many restaurants were involved in the study?

The researchers constructed the bar chart on the next page:



Use the bar chart to answer these questions:

What was the most popular dish?

What was the dish that was mentioned least by restaurants?

• A survey was carried out which asked a number of teenagers what their preferred social network site is. The results were as follows:

|--|

Social	Facebook	Twitter	Bebo	MySpace
Network Site				
Number of				
Teenagers				

How many teenagers took part in the study?

Represent this information on a bar chart (use graph paper).

Use the bar chart to answer these questions:

Can you make any conclusions about Facebook?

What was the least popular social network site? _____

• A number of adults were asked whether or not they liked or disliked maths. There was also an option to say they were unsure. Based on the information received, the following pie chart was drawn.



Using the pie chart above, answer the following questions:

How many adults took part in the study?

What sector was the greatest?

Why was the angle for "unsure" smaller than the other two angles?

• The unfinished pie chart below represents the males that took part in a study and the females that took part in a study.



The angle of 144° represents the males.

What angle represents the females?

• A number of couples were asked what their preferred type of accommodation would be. They were asked to choose between a bungalow, a two-story house and an apartment. The results were put into a frequency table.

Accommodation	Bungalow	2-Story House	Apartment
No. of Couples			

Complete the following table:

Accommodation	Number of Couples	Angle
Bungalow	3	$3 \times 30^{\circ} = 90^{\circ}$
2-Story House	7	7 x 30° = 210°
Apartment	2	2 x 30 = 60°
Total	12	360°

Construct a pie chart based on this information.

• Answer the following questions based on the pie chart constructed above.

How many couples took part in the survey?

What is the greatest angle?

What does this angle represent?

What conclusion can you make about these couples and what type of accommodation they would like to live in?

• A 4th class tutor decides to graph her students' attendance over the course of five weeks. There are 25 students in her class.



How many students were present in week 1?

How many students were present in week 2?

How many students were present in week 3?

How many students were present in week 4?

How many students were present in week 5?

Was there any time when the class was at full attendance?

Can you summarise the trend in the students' attendance?

• A shop that sells ice-cream is trying to decide whether it should have ice cream on sale all year round or just in the summer months. The shop decides to look at last years ice cream sales to help make their decision.

They firstly put the results into a frequency table:

Month	Jan	Feb	Mar	Apr	Мау	Jun	Jul	Aug	Sep	Oct	Nov	Dec
No. of	10	15	38	45	60	100	130	120	90	20	10	5
sales												

Construct a trend graph for this data (on graph paper) and then answer the following questions:

What month had the highest ice cream sales?

What month had the lowest ice cream sales?

Do you recommend any months were the shop should consider not selling ice cream? Why /Why not?

A number of women were asked how many times they get their hair cut a year. The results are represented on the following histogram.



Answer the following questions based on the histogram:

- a) How many women got their hair cut 0-2 times a year?
- b) How many women got their hair cut 2-4 times a year?
- c) How many women got their hair cut 4-8 times a year?
- d) How many women got their hair cut 8-12 times a year?

e) Complete the grouped frequency table based on the information you just calculated:

Number of hair	0-2	2-4	4-8	8-12
cuts				
Number of				
women				

- f) How many women took part in the survey?
- g) Can you draw any conclusions from the data about the number of times women get their hair cut during the year?

1. Based on the following information construct a histogram (use graph paper) and answer the question that follows.

A local shop wants to know the amount spent by customers in their shop on a given Saturday. They collect the following data.

Amount (€)	0-6	6-12	12-24	24-36	36-42
Number of	5	12	10	8	2
customers					

What conclusion, if any, can the shop make about the amount of money spent in the shop on that Saturday?

2. Based on the following information construct a grouped frequency table and a histogram (use graph paper).

The numbers in the box below represent the number of local GAA matches attended by a number of fans.

1	10	3	14	2	3	
17	4	9	2	8	5	
20	3	21	7	1	26	

Complete the grouped frequency table:

GAA matches	0-4	4-8	8-16	16-24	24-28
Number of fans					

Represent this information on a histogram:

1. Complete the following cumulative frequency table based on the grouped

frequency table.

Time (minutes)	0-20	20-40	40-80	80-100	100-120
No. of teenagers	12	15	8	6	1

Cumulative frequency table:

Time (in minutes)	<0	<20		
No. of teenagers	0			

2. Draw a cumulative frequency curve based on the completed cumulative frequency table.

1.	Work out the mean,	median and	mode for e	each of the	following	sets of
	numbers:					

i.	7	9	8	9	3		
Меа	an:						
Мес	dian:						
Ma	J						
MOG	ue:						
ii.	1	4	7	11	13	8	1
					-	-	
Меа	an:						
Мес	dian:						
Мос	de:						

Level 4	Resources			Mathematics	
iii. 0.2 Mean:	5.2	4.1	0.2	6.5	
Median:					

Mode:

-

iv. The figures below represent the number of phone calls received by the Gardai that report house burglaries over seven days.

Number of phone calls	Number of days
1	3
2	3
3	4
4	2
5	1

What is the mean number of phone calls the Gardai received?

What is the mode?

Resources Pack Level 4 Mathematics




Level 4: Mathematics Unit 1: Number

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Solution Sheet N1

Use your calculator to complete the following:

$$(a) 1.45 + 0.89 \times 4 = 1.45 + (3.56) = 5.01$$

$$(b) \frac{2}{7} \times \frac{1}{7} - 0.00035 = \frac{2}{49} - 0.00035 = -0.04047$$

$$(c) 2 - 3 \times 5 + 6 = 2 - (15) + 6 = -13 + 6 = -77$$

$$(d) 19.875 \div 4 + (2 \times 1) = (4.969) + (2) = 6.969$$

$$(e) \sqrt{289} - 3) \div 2 = (17 - 3) \div 2 = (17 - 3) \div 2 = -(14) \div 2 = -77$$

$$(g) \frac{4}{11} \text{ of } \notin 4.79 = \frac{4}{11} \times \notin 4.79 = -(1.74)$$

$$(h) 4(\pi) (3^{2}) = 4\pi (9) = 113.1$$

$$(i) \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} = \frac{2}{9} + \frac{1}{3} = \frac{5}{9} = -0.05$$

$$(j) \frac{2}{5} \times 1.75 - 0.75 = 0.7 - 0.75$$

$$(k) 279 \div \pi = \frac{88.81}{100}$$

$$(l) \frac{4}{11} \div 1.64 - 3 \times 0 = -(0.222) - (0) = 0.222$$

$$(m) 13\% \text{ of } 169 = \frac{13}{100} \times 169 = -21.97$$

$$(n) \sqrt{3^{2} + 16} = \frac{\sqrt{9 + 16}}{-5} = \sqrt{25}$$

Solution Sheet N2	2			
Question 1 Evaluate the follow	wing:			
(a) 14 x 12	168	(b) 23 x 16	368	
(c) 44 x 18	792	(d) 16 x 14	224	
(e) 21 x 11	231	(f) 32 x 34	1088	
(g) 18 x 45	810	(h) 7 x 32	224	
Question 2 Evaluate the follow	wing:			
(a) 256 ÷ 16	16	(b) 180 ÷ 18	10	
(c) 75 ÷ 25	3	(d) 272 ÷ 17	16	
(e) 450 ÷ 25	18	(f) 1152 ÷ 36	32	
(g) 4500 ÷ 90	50	(h) 1320÷ 12	110	

Question 3

Evaluate the following:(a) $23 \times 12 \div 3$ 92(b) $36 \times 18 \div 24$ (c) $690 \div 15 \times 12$ 552(d) $360 \div 48 \times 22$ (e) $32 \times 414 \div 18$ 736(f) $40 \times 165 \div 15$

Question 1

Convert the following percentages to fractions in their simplest form:



(b) 18%	9 50
(d) 35%	7 20
(f) 19%	19 10
(h) 40%	2 5
(j) 48%	12 25
() 11%	11 10
(n) 90%	9

50
$\frac{7}{20}$
19 100
2 5
$\frac{12}{25}$
11 100
9 10

Question 2

Evaluate the following:

(a) 20% of 100	20
(b) 30% of 50	15
(c) 5% of 200	10
(d) 14% of 500	70
(e) 90% of 200	180
(f) 17% of 150	25.5
(g) 26% of 560	145.6
(h) 72% of 340	244.8
(i) 60% of 900	540

Question 1

Write the following as fractions in their simplest form:



Question 2

Rewrite the following fractions as percentages:



Question 1

Convert the following decimals to fractions in their simplest form:

(a) 0.4	2 5	(b) 0.18	9 50
(c) 0.65	$\frac{13}{20}$	(d) 0.125	1 8
(e) 0.878	439 500	(f) 0.9	9 10
(g) 0.32	8 25	(h) 0.97	97 100
(i) 0.256	32 125	(j) 0.25	$\frac{1}{4}$
(k) 0.216	27 125	(I) 0.85	$\frac{17}{20}$
(m) 0.36	9 25	(n) 0.001	1 1000

Question 2

Convert the following decimals to percentages.

(a) 0.65	65%	(b) 0.3	30%
(c) 0.58	58%	(d) 0.365	36.5%
(e) 0.423	42.3%	(f) 0.08	8%
(g) 0.63	63%	(h) 0.158	15.8%
(i) 0.001	0.1%	(j) 0.09	9%
(k) 0.26	26%	(I) 0.525	52.5%
(m) 0.082	8.2%	(n) 1.0	100%

Calculate the gross income for the following people:

 Nancy is currently working as a substitute tutor. She gets paid by the hour. The government currently pays substitute tutors €28.50 per hour. Nancy worked 7.25 hours last week. What was her gross pay?

7.25 hours x 28.50 per hour = 206.625 Nancy's Gross Pay = **€206.63**

 John works with his father as an electrician. He works 37 hours a week and gets paid €9.10 an hour. Last week he had to work 4 hours overtime on Sunday and he gets paid time and half for overtime. What was John's gross pay last week?

Basic Pay: 37 hours x 9.10 per hour = 336.7 Overtime: 9.10 + 4.55 (time and a half rate of pay) = \in 13.65 per overtime hour

4 hours x 13.65 per hour = 54.6

John's Gross Pay = €336.7 + €54.6 = €391.30

Calculate the total deductions for the following people:

Keith has a gross pay of €367 and a tax credit of €75. He pays 2% Income Levy and 20% PAYE. What are Keith's total deductions?

Income Levy = 2% of Gross Pay = 2% of 367 = \in 7.34 Gross Pay – Tax Credit = taxable income \in 367 - \in 75 = \in 292 PAYE = 20% of \in 292 = \in 58.40 Total Deductions = \in 7.34 + \in 58.40 = \in 65.74 Maura has a gross pay of €515 and a tax credit of €104. She pays 2% Income Levy, 21% PAYE and 5.8% PRSI. She also contributes €1.90 to her Trade Union. What are Maura's total deductions?

Income Levy = 2% of Gross Pay = 2% of 515 = $\in 10.30$ Gross Pay – Tax Credit = taxable income $\in 515 - \in 104 = \in 411$ PAYE = 21% of $\in 411 = \in 86.31$ PRSI = 5.8% of $\in 411 = \in 23.84$ Trade Union subscription = $\in 1.90$ Total Deductions = $\in 10.30 + \in 86.31 + \in 23.84 + \in 1.90 = \in 122.35$

Calculate the following:

Tim works in a local factory. He has a net pay of €279.50. His total deductions were €98.75.
 What is his gross pay?

Gross pay – total deductions = Net pay Gross Pay = Net Pay + Total Deductions Gross pay = \notin 279.50 + \notin 98.75 Tim's Gross Pay is \notin 378.25

 Maeve is an auctioneer; she makes 1.7% commission on each house she sells. She sold a house this week for €155,000. What was her commission?

Michelle's commission is 1.7% of the sale price.

1.7% of €155,000 = $\frac{1.7}{100}$ x 155000 = 2,635

Michelle's commission is **€2,635**.

Jerry is a sales representative for a farm machine company. His monthly salary is €1,200. He earns 4.9% commission on any machinery he sells. In August he sold €24,580 worth of machinery. What was his gross salary in August? He pays 2% Income Levy, 20% PAYE and 6% PRSI. If his tax credit is €321, what was his net pay in August?

Jerry's monthly salary is €1,200

4.9% commission on €24,580: = $\frac{4.9}{100}$ x 24580 = 1,204.42 Jerry's gross pay in August = €1,200 + €1,204.42 = €2,404.42 Income Levy is 2% of gross pay = 2% of €2,404.42 = €48.09 Gross Pay – tax credit = taxable income €2,404.42 - €321 = €2,083.42

PRSI = 20% of €2,083.42 = €416.68 PAYE = 6% of €2,083.42 = €125

Total Deductions = €48.09 + €416.68 + €125 = €589.77

Net Pay = Gross Pay – Total Deductions

Net Pay = €2,404.42 - €589.77

Jerry's net pay in August = €1,814.65

- A strawberry picker earns 21 cent per punnet of strawberries picked. On a good day a strawberry picker picked 213 punnets of strawberries. How much did he earn?
 €0.21 per punnet
 €0.21 x 213 punnets = €44.73
- Thomas is an auctioneer. He charges 1.75% commission on the sale price of a house.
 Calculate Thomas's commission on the sale of a house for €115,850.

1.75% of 115,850 = **€2,027.38**

 Una runs a stationery shop in her local town. Her total sales for 2010 were €58,850. The cost of her goods was €23,987. Calculate Una's gross profit.

Una's total operating costs came to €16,107 in 2010. What was Una's salary?

Payments	58,850
Costs	23,987
Operating Costs	<u>16,107</u>
	18,756

- Gross profit = profit Costs
 Gross Profit = €58,850 €23,987 = €34,863
- (ii) Una's Salary was €18, 756 (amount left over after all expenses are paid)

Level 4

- Catherine is retiling her kitchen. She has hired Peter to do this job. He gave her the following estimate for the job:
 - 9 hours at €9.50 an hour
 - 6 boxes of tiles @ €42.15 per box

3 tubs of tile adhesive @ €6.20 per tub

Fitting: 12m² this costs €12.49

VAT @ 13.5%

When the job is finished there is one box of tiles left over and 0.75 of a tube of adhesive left over.

- (i) Calculate the total cost of the job
- (ii) Calculate the total cost of wastage

(i) Labour Cos	sts:	9 hours x 9.50 per hour	€ 85.50
Material Costs 6 boxes x €42		42.15 per box	€252.90
	3 tubs x €6.	20 per tub	€ 18.60
	Fitting		€ 12.49
Total Costs (ex v	at)		€369.49
Vat @ 13.5 %	<u>(13.5% of</u> €	<u>369.49)</u>	€ 49.88
Total Costs (inc v	vat)		€419.37
(ii) Wastage:			
Material waste	1 box x €42	.15	€42.15
	<u>0.75 tub x</u> €	6.20	€ 4.65
			€46.80

Total cost of wasted material is €46.80

 Joe is a dairy farmer. He received €97,650 in payments in 2009. His operating expenses are as follows:

Equipment	€24,560	Light and heat	€1,243
Transport	€4,300	Fertiliser	€3,250
Feeding	€3,940	Veterinary Bills	€2,875

(i) Draw up Joe's profit and loss account.

(ii) What is Joe's gross salary if he reinvests 12% of his profits?

(i) Total Payments		€97,650
Cost of Equipment		<u>€ 24,560</u>
	Gross Profit	€73,090
Operating Costs:		
Transport		€4,300
Feeding		€3,940
Light & Heat		€1,243
Fertiliser		€3,250
Vet. Bills		<u>€2,875</u>
		€15,608
		<u>€15,608</u>
	Profit:	€57,482

(iii) Joe's operating profit is €57,482. He reinvests 12% of this:
12% of €57,482= €6,897.84

Therefore Joe's retained salary is €57,482 - €6,897.84 = €50,584.16

Level 4		Resources		Mathematics
Solution Sheet N8				
Question 1 Simplify the followin	g ratios (if po	ssible):		
(a) 14:7	2:1		(b) 16:4	4:1
(c) 11:22	1:2		(d) 18:6	3:1
(e) 2:1	2:1		(f) 15:5	3:1
(g) 4:12	1:3		(h) 24:6	4:1
(i) 3:18	1:6		(j) 12:48	1:4

Question 2

Write the following ratios in fraction form:

(a) 1:2	¹ / ₂	(b) 1:4	¹ / ₄
(c) 2:4	¹ / ₂	(d) 6:18	¹ / ₃
(e) 2:3	² / ₃	(f) 11:22	¹ / ₂
(g) 4:2	2/1	(h) 16:4	⁴ / ₁
(i) 15:5	3/1	(j) 13:10	¹³ / ₁₀

Question 1

The following table represents Euro exchange rates:

Currency	We Sell	We Buy
Sterling	0.835	0.8586
US Dollars	1.3406	1.3786
Australian Dollars	1.3633	1.4019
Thai Baht	40.0648	41.2024
Swiss Franc	1.3202	1.3576
Norwegian Krone	8.033	8.261
Indian Rupee	60.7154	62.4394

Source: <u>www.aib.ie</u> (16th November 2010)

Using the table above convert the following:

- (a) €25 to Australian Dollars
- (b) €100 to Indian Rupee
- (c) €10 to Thai Baht
- (d) £50 Sterling to Euro
- (e) 40 US Dollars to Euro
- (f) €36 to Norwegian Krone
- (g) €57 to Swiss Franc
- (h) £10 Sterling to US Dollars
- (i) 20 American Dollars to Indian Rupee
- (j) 50 Australian Dollars to Thai Baht
- (k) 15 Norwegian Krone to Swiss Franc

 Aus \$34.08

 6071.54 Indian Rupee

 400.65 Baht

 €58.23

 €29.01

 289.19 Norwegian Krone

 75.25 Swiss Franc

 \$15.61

 880.98 Indian Rupee

 1429.11 Baht

2.40 Swiss Franc

* Hint: For the last 4 questions convert the money to euro first, round your answers to two decimal places and then convert from euro to the required currency.

Question 1

Change the following numbers into scientific notation.

(a) 1000	1.0 x 10 ³	(b) 3890	3.89 x 10 ³
(c) 5,000,000	5.0 x 10 ⁶	(d) 4,500,000	4.5 x 10 ⁶
(e) 670,000	6.7 x 10 ⁵	(f) 56,789	5.6789 x 10 ⁴
(g) 5,342	5.342 x 10 ³	(h) 69,534,000	6.9534 x 10 ⁷
(i) 78,901	7.8901 x 10 ⁴	(j) 4,670,000	4.67 x 10 ⁶
(k) 0.00056	5.6 x 10 ⁻⁴	(I) 0.00001	1.0 x 10 ⁻⁵
(m) 0.23	2.3 x 10 ⁻¹	(n) 0.00605	6.05 x 10 ⁻³

Question 2

Change the following numbers from scientific notation into standard form



Question 1

Round off the following to **<u>one</u>** decimal place.

(a) 3.66	3.7	(b) 4.91	4.9
(c) 8.756	8.8	(d) 13.487	13.5
(e) 11.608	11.6	(f) 16.182	16.2
(g) 23.478	23.5	(h) 36.828	36.8
(i) 71.419	71.4	(j) 8.673	8.7

Question 2

Round off the following to **two** decimal places.

(a) 1.653	1.65	(b) 8.592	8.59
(c) 15.6395	15.64	(d) 11.5937	11.59
(e) 27.21593	27.22	(f) 95.6389	95.64
(g) 5.6926	5.69	(h) 32.6512	32.65
(i) 26.95341	26.95	(j) 11.52392	11.52

Question 3

Round off the following to $\underline{\textbf{three}}$ decimal places

(a) 18.65239	18.652	(b) 1.23698	1.237
(c) 13.958633	13.959	(d) 52.63231	62.632
(e) 19.53926	19.539	(f) 56.36825	56.368
(g) 11.23897	11.239	(h) 12.3659	12.366

Question 1

Calculate the percentage error between the estimate and the actual value of the different items below:

Item	Estimate	Actual Value	Percentage Error
Box of Cereal	€2.50	€3.20	21.88%
Adidas Runners	€40	€55	27.27%
HD Television	€500	€480	4.17%
GAA All Ireland	€56	€70	20%
Final Ticket			
16GB Ipod Nano	€200	€170	17.65%
Nintendo Wii	€185	€200	7.5%
Console			
XFactor Live Tour	€45	€54	16.67%
Tickets			
Pre-owned Nissan	€3,500	€3,750	6.67%
Micra (2003 model)			

Resources

Solution Sheet N13

Without using your calculator simplify the following:

- (a) $3^{2} \times 3^{7}$ $3^{2+7} = 3^{9}$ (b) $4^{3} \cdot 4^{11} = 4^{3+11} = 4^{14}$ (c) $3^{7} \div 3^{2}$ $3^{7-2} = 3^{5}$ (d) $(3^{7})^{3} = 3^{7(3)} = 3^{21}$ (e) $(3x)^{3} = (3^{3}x^{3}) = 27x^{3}$ (f) $5^{9} = 1$ (g) $7^{4} \times (7^{3} \cdot 7^{2})$ (h) $(12a)^{4} = (12)^{4}(a^{4})$ $7^{3+2} = 7^{5}$ $7^{4} \cdot 7^{5} = 7^{9}$ $= 20,736a^{4}$ (j) $(x^{a})^{3} = x^{3a}$ (j) $7^{4} \div (7^{3} \cdot 7^{2}) = 7^{4} \div (7^{3-2}) = 7^{4}$ $= 7^{4-1} = 7^{3}$
- (k) $13^4 \div (13^1 \cdot 13^3) = 13^4 \div (13^{1+3})$ = $13^4 \div 13^4$ = $13^{4-4} = 13^0$ = 1

(j)
$$7^4 \div (7^3 \cdot 7^2) = 7^4 \div (7^{3-2}) = 7^4 \div 7^1$$

= $7^{4-1} = 7^3$
(l) $8^6 \times 8^{-2} = 8^{6+(-2)}$
= 8^4

(m)
$$8^6 \div 8^{-2} = 8^{6-(-2)}$$

= 8^8
(n) $13^{10} \div (13^1 \div 13^3) = 13^{10} \div (13^{1-3})$
= $13^{10} \div 13^{-2}$
= $13^{10-(-2)}$
= 13^{12}

Without using your calculator simplify the following:

 $= \log_4 192$

(a)
$$\log_4 3 + \log_4 5 = \log_4 3(5)$$

 $\log_4 15$
(b) $\log_4 24 - \log_4 3 = \log_4 \frac{24}{3}$
 $\log_4 8$
(c) $\log_3 \frac{1}{5} + \log_3 5 = \log_3 (\frac{1}{5})(5)$
 $= \log_3 1$
 $= \log_7 8$
(e) $\log_8 8 = 1$
(f) $\log_4 8 + \log_4 \frac{1}{2} = \log_4 8(\frac{1}{2})$
 $= \log_4 8(\frac{1}{2})$
 $= \log_4 4$
 $= 1$
(g) $-\log_4 6 + \log_4 42 = \log_4 42 - \log_4 6$
 $= \log_4 \frac{42}{6}$
 $= \log_4 7$
(i) $-\log_8 6 + \log_8 48 = \log_8 48 - \log_8 6$
 $= \log_8 \frac{48}{6}$
 $= \log_8 8 = 1$
(k) $\log_9 \frac{x}{2} = \log_9 x - \log_9 2$
(l) $\log_4 4^2 + \log_4 12 = \log_4 16 + \log_4 12$
 $= \log_4 16(12)$
 $= \log_4 10(12)$
 $= \log_4 10(12)$
(b) $\log_4 1 + \log_4 3 = \log_4 16 + \log_4 12$
(c) $\log_1 0 + \log_4 1 + \log_4 12$
(c) $\log_1 0 + \log_4 1 + \log_4 12$
 $= \log_4 16(12)$
 $= \log_4 192$
(c) $\log_1 0 + \log_4 1 + \log_4 1 + \log_4 12$
(c) $\log_1 0 + \log_4 1 + \log_4 1 + \log_4 12$
(c) $\log_1 0 + \log_4 1 + \log_4 1 + \log_4 12$
(c) $\log_1 0 + \log_4 1 + \log_4$

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Express the following as indices:

(a)
$$\log_{17} 83521 = 4$$
 17⁴ = 83,521 (b) $\log_{10} 560 = 2.748$ 10^{2.748} = 560

(c)
$$\log_5 0.008 = -3$$
 $5^{-3} = 0.008$ (d) $\log_{0.5} 0.25 = 2$ $0.5^2 = 0.25$

Express the following as logs:

(e)
$$4^3 = 64$$
 $\log_4 64 = 3$ (f) $13^4 = 28,561$ $\log_{13} 28,561 = 4$
(g) $3^5 = 243$ $\log_3 243 = 5$ (h) $154^{0.5} = 12.41$ $\log_{154} 12.41 = 0.5$

Resources

Use Logs and Indices to solve the following:

(i)
$$2^{x} = 64$$
 $\log_{10} 2^{x} = \log_{10} 64$ (j) $\log_{3} 729 = x$ $3^{x} = 729$
 $x \log_{10} 2 = 1.806$ $\log_{10} 3^{x} = \log_{10} 729$
 $x (0.301) = 1.806$ $x \log_{10} 3 = 2.863$
 $x = \frac{1.806}{0.301} = 6$ $x (0.477) = 2.863$
 $x = \frac{2.863}{0.477} = 6$

(k) $3^{x} = 1.732$ $\log_{10} 3^{x} = \log_{10} 1.732$ (l) $\log_{6} 4.193 = x$ $6^{x} = 4.193$ $x \log_{10} 3 = 0.239$ $\log_{10} 6^{x} = \log_{10} 4.193$ x (0.477) = 0.239 $x = \frac{0.239}{0.477} = 0.5$ x (0.778) = 0.623 $x = \frac{0.623}{0.778} = 0.8$

(m) $8^x = 3.784$	$\log_{10} 8^x = \log 3.784$	(n) $\log_{10} 156 = x$	x = 2.193	
	$x \log_{10} 8 = 0.578$ $x (0.903) = 0.578$ $x = \frac{0.578}{0.903} = 0.64$			

Calculate the simple interest which is paid on the following loans:

(a)P = €450	$I = \frac{R}{100} x P I = \frac{5}{100} x 450 = 0.05 \times 450 = \textbf{€22.50}$
R = 5% T = one year	
(b) P = €6,400 R = 4.8% T = one year	$I = \frac{R}{100} x P I = \frac{4.8}{100} x 6400 = 0.048 \times 6400 = \textbf{€307.20}$
(c)P = €12,750 R = 3.5% T = Two yea	I = $\frac{3.5}{100}x12750$ = 0.035 x 12750 = €446.25 €446.25 each year, therefore the interest is €446.25 + €446.25 €892.50
(d) P = €23,650 R = 2.75% T = Three years	I = $\frac{2.75}{100}x23650 = 0.0275 \times 23650 = €650.38$ €650.38 each year, therefore the interest is €650.38 €650.38 + €650.38

€1,951.14

Level 4

Resources

Calculate the simple interest which is earned on the following savings:

(e) P = €859 I =
$$\frac{6.2}{100}x859 = 0.062 \times 859 = €53.26$$

R = 6.2% T = one year

$R = 3.6\%$ $\in 225$ each year, therefore the interest is $\in 22$ $T = two years$ $+ \in 22$	(f) P = €6,250	I = $\frac{3.6}{100}$ x 6250 = 0.036 x 6250 = €225	
	R = 3.6% T = two years	€225 each year, therefore the interest is <u>+</u>	€225 €225

€450

(g) P = €5,300 I =
$$\frac{3.5}{100}x5300 = 0.035 \times 5300 = €185.50$$

R = 3.5%	€185.50 each year, therefore the interest is	€185.50
T = Two years	:	<u>+ €185.50</u>

€371

		€130.35
	<u>+</u>	€43.45 <u>€43.45</u>
R 2.75% T = Three years	€43.45 each year, therefore the interest is	€43.45
(h) =€1,580	$I = \frac{2.75}{100} x 1580 = 0.0275 \times 1580 =$	€43.45

Level 4		Resources	Mathematics
Calculate the a (i) P = €895 I = €135	Mount (A): A = P + I A = €895 + 135 A = €1,030	(j) P = €1264 I = €524	A = P + I A = €1264 + €524 A = €1,788

(k) P = €2,365	A = P + I	(I) P = €1,583	A = P + I
I = €314	A = €2365 + €314	I =€76	A = €1583 + €76
	A = €2679		A = €1,659

Calculate the compound interest p.a. which is paid on the following loans:

(a)P = €1,650
$$A = P\left(1 + \frac{R}{100}\right)^n$$
 $A = 1650\left(1 + \frac{3}{100}\right)^3 = 1650(1 + 0.03)^3$
R = 3% $= 1650(1.03)^3$
T = 3 years $= 1650(1.093)$
A = €1,803.45

Amount – Principal = Interest €1,803.45 - €1,650 = €153.45

(b) P = €5,475
$$A = P\left(1 + \frac{R}{100}\right)^n$$
 $A = 5475\left(1 + \frac{4.8}{100}\right)^5 = 5475(1 + 0.048)^5$
R = 4.8%
T = 5 years $= 5475(1.264)$
A = €6,920.40

Amount – Principal = Interest €6,920.40 - €5,475 = €1,445.40

(c)
$$P = \notin 12,750$$

 $R = 3.5\%$
 $T = 4 \text{ years}$
 $A = 12750 \left(1 + \frac{3.5}{100}\right)^4 = 12750 \left(1 + 0.035\right)^4$
 $= 12750 \left(1.035\right)^4$
 $= 12750(1.148)$
 $A = \notin 14,637$

Amount – Principal = Interest €14,637 - €12,750 = €1,887

Level 4

Calculate the compound interest which is earned on the following savings:

Once interest is compounded more than once a year r is affected by the number of times interest is applied and n is the number of time periods.

(d) P = €5,890 Interest is applied monthly therefore $R = 3\% \div 12 = 0.25\%$ monthly R = 3%and n = 12 times a year 1 x 12 = 12 time periods. $A = P \left(1 + \frac{R}{100} \right)^n A = 5890 \left(1 + \frac{0.25}{100} \right)^{12} = 5890 \left(1 + 0.0025 \right)^{12}$ T = one year $= 5890 (1.0025)^{12}$ Compounded monthly = 5890(1.03) A= €6,066.70 Amount – Principal = Interest €6,066.70 - €5,890 = €1,76.70 (e) P = €6,250 Compounded bi-annually R = 3.6% $r = 3.6\% \div 2 = 1.8\%$ bi-annually and $n = 2 \times 2 = 4$ time periods $A = 6250 \left(1 + \frac{1.8}{100} \right)^4 = 6250 \left(1 + 0.018 \right)^4$ T = two years $= 6250 (1.018)^4$ = 6250(1.074)A=€6,712.50 Amount – Principal = Interest €6,712.50 - €6,250 = €462.50 (f) P = €5,300 Compounded quarterly R = 3.5% $r = 3.5\% \div 4 = 0.875\%$ quarterly and $n = 2 \times 4 = 8$ time periods $A = 5300 \left(1 + \frac{0.875}{100} \right)^8 = 5300 \left(1 + 0.00875 \right)^8$ T = two years $= 5300 (1.00875)^{8}$ = 5300(1.072) A= €5,681.60

> Amount – Principal = Interest €5,681.60 - €5,300 = €381.60

Resources

(g) €890 is invested for 4 years at 4.5% p.a. compounded interest. Interest is compounded biannually. What is the amount at the end of the time period?

 $r = 4.5\% \div 2 = 2.25\%$ bi-annually and $n = 4 \times 2 = 8$ time periods

A = 890
$$\left(1 + \frac{2.25}{100}\right)^8$$
 = 890 (1 + 0.0225)⁸
= 890(1.0225)⁸
= 890(1.195) A= €1,063.55

(h) What principal amount (P) will become €6,000 in two years when it is invested at 3%p.a. compounded interest. Interest is compounded quarterly.

 $r = 3\% \div 4 = 0.75\%$ quarterly and $n = 2 \times 4 = 8$ time periods

$$6,000 = P\left(1 + \frac{0.075}{100}\right)^{8}$$

$$6,000 = P(1.0075)^{8}$$

$$6,000 = P(1.062)$$

$$P = \left(\frac{6,000}{1.062}\right)$$

P=€5,649.72

(i) Which costs the borrower more: a simple interest loan of €3,000 for two years at 3.1% simple interest or a compounded interest loan of €3,000 over two years at 2.9% p.a. compound interest which is compounded monthly?

Simple Interest Loan:

I = $\frac{R}{100}xP$ I = $\frac{3.1}{100}x3000$ = 0.031 x 3000 = €93 each year

For two years the interest would be €186.

Compound Interest Loan:

 $r = 2.9\% \div 12 = 0.242\%$ quarterly and $n = 2 \times 12 = 24$ time periods

A =
$$3000 \left(1 + \frac{0.242}{100}\right)^{24}$$
 = 3000 (1 + 0.0024)2²⁴
= 3000 (1.0024)²⁴
= 3000 (1.06) A= €3,180

Interest = Amount – Principal = €3,180 - €3000 = €180

Therefore it is cheaper to take out the €3,000 loan with 2.9% p.a. compound interest which is compounded monthly.



Solution Sheets Level 4: Mathematics Unit 2: Geometry

In the space on the right name each of the following shapes.





State if each figure below has folding symmetry. If it has, then indicate the line that gives this symmetry.











(a) Plot the points (2, 3), (1, 4), (3, 1)



(b) Plot the points (-2, 3), (-1, 4), (3, -1)



(c) What are the coordinates of the points in the diagram below?



(d)





(e) A is the point (1, 3). B is the point (3, 1). Label these points in the diagram below:





From the graph the baby will take six weeks to reach a weight of eleven pounds.

• From Pythagoras

$$a2 = b2 + c2$$
$$a2 = 42 + 72$$
$$a2 = 16 + 49$$
$$a2 = 65$$
$$a = \sqrt{65}$$
$$a = 8.06cm$$

• Similarly

$$a^{2} = b^{2} + c^{2}$$

$$5.6^{2} = b^{2} + 4^{2}$$

$$b^{2} + 4^{2} = 5.6^{2}$$

$$b^{2} + 16 = 31.36$$

$$b^{2} = 31.36 - 16$$

$$b^{2} = 15.36$$

$$b = \sqrt{15.36}$$

$$b = 3.92cm$$

• The sides of the rectangle form a right-angled triangle so

$$a^{2} = b^{2} + c^{2}$$

$$a^{2} = 5.34^{2} + 2.78^{2}$$

$$a^{2} = 28.5156 + 7.7284$$

$$a^{2} = 36.244$$

$$a = \sqrt{36.244}$$

$$a = 6.02cm$$

• Distance

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3 - (-2))^2 + ((-4) - 5)^2}$$
$$= \sqrt{(5)^2 + (-9)^2}$$
$$= \sqrt{25 + 81}$$
$$= \sqrt{106}$$
$$= 10.30$$

• The radius of the circle is the distance between the two points.

$$r = \sqrt{(7-1)^2 + (-10-3)^2}$$

= $\sqrt{(6)^2 + (-13)^2}$
= $\sqrt{36+169}$
= $\sqrt{205}$
= 14.32

• The length of the diagonal is

$$d = \sqrt{(2-5)^2 + (3-1)^2}$$

= $\sqrt{(-3)^2 + 2^2}$
= $\sqrt{9+4}$
= $\sqrt{13}$
= 3.61
• Use the mid-point formula:

$$c = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{2 + 8}{2}, \frac{6 + 10}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{16}{2}\right)$$
$$= (5, 8)$$

• The centre of a circle is the mid-point of a diameter.

$$c = \left(\frac{-7+3}{2}, \frac{8-6}{2}\right)$$
$$= \left(\frac{-4}{2}, \frac{2}{2}\right)$$
$$= (-2, 1)$$

The diagonals of a square bisect each other. The two diagonals are the line segment joining (0, 0) to (4, 4) and the line segment joining (0, 4) to (4, 0). You can find the midpoint of either of these and it will get you the same point.

$$c = \left(\frac{0+4}{2}, \frac{0+4}{2}\right)$$
$$= (2,2)$$
or
$$c = \left(\frac{0+4}{2}, \frac{4+0}{2}\right)$$
$$= (2,2)$$

• Use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 6}{8 - 2}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

• Use the slope formula:

$$m = \frac{-6 - 8}{3 + 7}$$
$$= \frac{-14}{10}$$
$$= -\frac{7}{5}$$

• The fractions mean that you should multiply above and below the fraction line by the lowest common denominator. You can do this at the start or the end.

$$m = \frac{\frac{3}{5} - 1}{-3 - \frac{3}{2}}$$
$$= \frac{10\left(\frac{3}{5} - 1\right)}{10\left(-3 - \frac{3}{2}\right)}$$
$$= \frac{6 - 10}{-30 - 15}$$
$$= \frac{-4}{-45}$$
$$= \frac{4}{45}$$

• This means solving for *y* instead of *m*.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{y_2 - y_1}{x_2 - x_1} = m$$
$$\frac{y - 8}{1 - 4} = 3$$
$$\frac{y - 8}{-3} = 3$$
$$y - 8 = -3 \times 3$$
$$y - 8 = -9$$
$$y = -9 + 8$$
$$y = -1$$

• First write
$$-\frac{2}{3}$$
 as $\frac{-2}{3}$ to get

$$m = \frac{\frac{4}{7} - \frac{4}{3}}{\frac{-2}{3} - \frac{5}{6}}$$
$$= \frac{42\left(\frac{4}{7} - \frac{4}{3}\right)}{42\left(\frac{-2}{3} - \frac{5}{6}\right)}$$
$$= \frac{24 - 56}{-28 - 35}$$
$$= \frac{-32}{-63}$$
$$= \frac{32}{63}$$
$$= 0.508$$

- The slope of AB is $m_1 = \frac{5+3}{-2-1} = -\frac{8}{3}$. The slope of CD is $m_2 = \frac{-9-2}{-7-3} = \frac{11}{10} \neq m_1$ so AB is not parallel to CD.
- The opposite sides of a parallelogram are parallel. We use this as follows:

The slope of the line joining (-1, -4) to (2, 3) is $m_1 = \frac{3+4}{2+1} = \frac{7}{3}$ The slope of the line joining (2, 3) to (4, 6) is $m_2 = \frac{6-3}{4-2} = \frac{3}{2}$ The slope of the line joining (4, 6) to (1, -1) is $m_3 = \frac{-1-6}{1-4} = \frac{-7}{-3} = \frac{7}{3}$ The slope of the line joining (1, -1) to (-1, -4) is $m_4 = \frac{-4+1}{-1-1} = \frac{-3}{-2} = \frac{3}{2}$ $m_1 = m_3$ and $m_2 = m_4$ which means that the opposite sides are parallel. The figure is a parallelogram.

• The slope of AB is $m_1 = \frac{-3-1}{-1-1} = \frac{-4}{-2} = 2$ The slope of BC is $m_2 = \frac{1+3}{-\frac{1}{2}+1} = 8$ The slope of CD is $m_3 = \frac{3-1}{\frac{1}{2}+\frac{1}{2}} = 2$ The slope of DA is $m_4 = \frac{3-1}{\frac{1}{2}-1} = -4$

This means that AB is parallel to CD but BC is not parallel to DA so the figure ABCD is not a parallelogram.

Level 4

Resources

Solution Sheet G10

• The slope of AB is $m_1 = \frac{4+3}{-2-1} = \frac{7}{-3} = -\frac{7}{3}$. If two lines of slope m_1 and m_2 are perpendicular

then $m_1 \times m_2 = -1$. Hence

$$\frac{7}{3} \times m_2 = -1$$
$$m_2 = -1 \times \left(-\frac{3}{7}\right)$$
$$m_2 = \frac{3}{7}$$

- The slope of AB is $m_1 = \frac{3-1}{2-1} = \frac{2}{1} = 2$
 - The slope of CD is $m_2 = \frac{-1-1}{5-6} = \frac{-2}{-1} = 2 = m_1$. AB is parallel to CD.
 - The slope of BC is $m_3 = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$

The slope of AD is $m_4 = \frac{-1-1}{5-1} = \frac{-2}{4} = -\frac{1}{2} = m_3$. BC is parallel to AD.

The opposite sides are parallel so the figure is a parallelogram.

 $m_1 \times m_3 = 2 \times \left(-\frac{1}{2}\right) = -1 = m_2 \times m_4$. The adjacent sides are perpendicular so the figure is a

rectangle.

• The slope of AB is $m_1 = \frac{3-6}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$

The perpendicular bisector of AB will have a slope

$$m_1 \times m_2 = -1$$
$$\frac{1}{2} \times m_2 = -1$$
$$m_2 = -1 \times \left(\frac{2}{1}\right)$$
$$m_2 = -2$$

Level 4

Resources

Mathematics

Solution Sheet G11

• The slope of AB is $m_1 = \frac{8+3}{-2-1} = \frac{11}{-3} = -\frac{11}{3}$.

A point on AB is (1, -3). The equation of the line AB is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -\frac{11}{3}(x - 1)$$

$$3(y + 3) = -11(x - 1)$$

$$3y + 9 = -11x + 11$$

$$11x + 3y = 2$$

or

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{11}{3}(x + 2)$$

$$3(y - 8) = -11(x + 2)$$

$$3y - 24 = -11x - 22$$

$$11x + 3y = 2$$

• The midpoint, C, of the line segment PQ is $\left(\frac{-8+6}{2}, \frac{7-3}{2}\right) = (-1,2)$

The slope of PQ is $m_1 = \frac{7+3}{-8-6} = \frac{10}{-14} = -\frac{5}{7}$

The slope of the perpendicular through C is $m_2 = \frac{7}{5}$. The equation of the perpendicular line is

> $y-2 = \frac{7}{5}(x+1)$ 5(y-2) = 7(x+1) 5y-10 = 7x+7 5y = 7x+17

• Find the point of intersection of the lines 3x+4y = 18 and 2x - 5y = -11

The equation of the *x*-axis is y = 0. We substitute this value of *y* into the equation to get $2x - 0 = 8 \Rightarrow x = 4$. The line meets the *x*-axis at the point (4, 0).

• We solve between the two equations to get the common point.

$$3x + 4y = 18$$
$$2x - 5y = -11$$
$$15x + 20y = 90$$
$$8x - 20y = -44$$
$$23x = 46$$
$$x = 2$$
$$3(2) + 4y = 18$$
$$6 + 4y = 18$$
$$4y = 12$$
$$y = 3$$

This means that the lines meet at (2, 3).

Level 4

Solution Sheet G12

• Equation of the circle is

$$x^{2} + y^{2} = r^{2}$$

$$x^{2} + y^{2} = 13^{2}$$

$$x^{2} + y^{2} = 169$$
• First write the circle as $x^{2} + y^{2} = r^{2}$.
$$x^{2} + y^{2} = 225$$

$$x^{2} + y^{2} = 15^{2}$$

which means that the radius is 15.

• The first thing to do is get the radius of the circle. The radius of the circle will be the distance between the points (0, 0) and (3, -4).

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$r = \sqrt{(3 - 0)^2 + (-4 - 0)^2}$$

$$r = \sqrt{9 + 16}$$

$$r = \sqrt{25}$$

$$r = 5$$

We use this value of the radius in the equation of the circle to get

$$x2 + y2 = 52$$
$$x2 + y2 = 25$$

• We divide both sides of the equation by 16 to bring the equation into the form we use normally.

$$16x^{2} + 16y^{2} = 289$$
$$x^{2} + y^{2} = \frac{289}{16}$$
$$x^{2} + y^{2} = \left(\frac{17}{4}\right)^{2}$$

This means that the radius is $\frac{17}{4} = 4.25$.

Resources

• First we must get the point of intersection of the two lines.

$$x + 3y = -1$$
$$2x + y = 1$$
$$-2x - 6y = 2$$
$$2x + y = 1$$
$$-5y = 3$$
$$y = -\frac{3}{5}$$
$$x + 3\left(-\frac{3}{5}\right) = -1$$
$$x = \frac{9}{5} - 1$$
$$x = \frac{4}{5}$$

The radius is now obtained by

$$r = \sqrt{\left(\frac{4}{5} - 0\right)^{2} + \left(-\frac{3}{5} - 0\right)^{2}}$$
$$r = \sqrt{\frac{16}{25} + \frac{9}{25}}$$
$$r = \sqrt{\frac{25}{25}}$$
$$r = 1$$

which means that the circle is

$$x2 + y2 = 12$$
$$x2 + y2 = 1$$

• Equation of the tangent is

$$xx_1 + yy_1 = r^2$$
$$x(3) + y(-4) = 25$$
$$3x - 4y = 25$$

• After you get the tangent you will have to simplify the equation.

$$x\left(\frac{3}{5}\right) + y\left(-\frac{4}{5}\right) = 1$$
$$\frac{3x}{5} - \frac{4y}{5} = 1$$
$$3x - 4y = 5$$

• The radius of the circle is 13 so the tangent is $xx_1 + yy_1 = 169$. This means that the tangent must meet the circle at the point (5, 12). The radius of the circle is perpendicular to the tangent.

$$m \times m_{1} = -1$$

$$\left(-\frac{5}{12}\right) \times m_{1} = -1$$

$$m_{1} = -1 \times \left(-\frac{12}{5}\right)$$

$$m_{1} = \frac{12}{5}$$

We use this value of the slope to get the equation of the radius.

$$y - y_1 = m(x - x_1)$$

$$y - 12 = \frac{12}{5}(x - 5)$$

$$5(y - 12) = 12(x - 5)$$

$$5y - 60 = 12x - 60$$

$$5y = 12x$$

Use your ruler to draw a line 5cm long. Label this AB. Adjust the radius of your compass to 3cm. With centre at A draw an arc. Adjust the radius of the compass to 4cm. With centre at B draw an arc to intersect the previous arc. Where the two arcs meet mark the point as C. Join AC and BC.



Use your ruler to draw the segment AB 6cm long. Use your protractor to draw a line at an angle of 43° to AB at the point A. Adjust the radius of your compass to be 8cm. With centre at B draw an arc to intersect the line from A. Mark this intersection point as C. Join AC and BC.



Draw the line AC 4cm long. With the protractor at A draw the angle 52°. Now put the protractor at C and draw an angle of 75°. Where the lines from the two angles intersect mark the point B.



Since the three angles in a triangle must add up to 180° you can calculate that |∠BCA|=180°-90°-50°=40°. Draw the line segment |BC| = 5cm. At the point *B* draw an angle of 50°. At the point *C* draw an angle of 40°. These two lines meet at the point *A*.



Draw the segment |*AB*| = 5cm. Using your set squares or protractor draw a vertical line at *A*. With your compass mark the point *D* 3cm above *A*. Draw a line perpendicular to *AD* at *D*. Use your compass to mark *C* which is 5cm from *D*. Join *ABCD*.



• We use the formula for the area of a rectangle.

$$A = l \times b$$
$$A = 4 \times 7$$
$$A = 28 cm^{2}$$

 We break this problem down into the straight edge part and the semi-circular parts. The straight edges are 100 + 100 = 200m.

The curved edges are $2 \times \frac{1}{2} \times 2\pi r = 2 \times 3.14 \times 25 = 157$.

The total length of track is 200 + 157 = 357m.

 A disc is just a filled in circle. Since the disc is made from all the metal that formed the rectangle then the area of the disc is equal to the area of the rectangle. It will be easier to convert the lengths to centimetres.

$$A = l \times b$$

$$A = 0.5m \times 1m$$

$$A = 50cm \times 100cm$$

$$A = 5000cm^{2}$$

We put this equal to the area of a disc (circle) and solve for the radius.

$$\pi r^{2} = 5000$$

$$r^{2} = \frac{5000}{\pi}$$

$$r^{2} = \frac{5000}{3.14}$$

$$r^{2} = 1592.36$$

$$r = \sqrt{1592.36}$$

$$r = 39.9cm$$

• One side of the triangle is horizontal and the other is vertical. This means that it is a rightangled triangle.



The area of a triangle is $\frac{1}{2}b \times h$ so the area of wall requiring paint is

$$A = \frac{1}{2} \times 2.2 \times 3.4$$
$$A = 3.74m^2$$

 Since the field is a rectangle the sides are at right angles to one another. Pythagoras' Theorem can be used to calculate the length of the diagonal.

$$l^{2} = 76^{2} + 54^{2}$$

$$l^{2} = 5776 + 2916$$

$$l^{2} = 8692$$

$$l = \sqrt{8692}$$

$$l = 93.23$$

The fence will be 93.23m long.

Draw a line segment AB 5.3cm long. Extend it at A. Draw an angle of 68° at the point A. Draw an angle of 35° at the point B. Where the lines from these angles meet is C, the top of the house. Use your set square to draw a perpendicular line to meet BA extended at the point D.



Measure the length |CD|. The correct answer is 5.18cm but if you get about 5.2cm that is accurate enough. This means that the wall is about 5.2m high.

Resources

• The diameter of the circle is 8cm which means that the radius is 4cm. The circumference is

$$c = 2\pi r$$

$$c = 2 \times 3.14 \times 4$$

$$c = 25.12cm$$

This is the **breadth of the rectangle**.

The area of tin needed for one can is the area of the rectangle added to the two ends. The area of the rectangle is

$$A_1 = l \times b$$
$$A_1 = 12 \times 25.12$$
$$A_1 = 301.44 cm^2$$

Each end has an area given by

$$A_2 = \pi r^2$$
$$A_2 = 3.14 \times 4^2$$
$$A_2 = 50.24 \, cm^2$$

The total area of a can is $A = 301.44 + 2 \times 50.24 = 401.92 cm^2 = \frac{401.92}{10000} m^2$

For 10000 cans $401.92m^2$ of tin is needed.



Level 4: Mathematics

Unit 3: Algebra

Question 1

To play table tennis in the local club, you pay an admission charge of €4 and an additional charge of \in 1.50 per hour. The total cost (C) of playing table tennis when t is time in hours, can be expressed by C = 1.5t + 4. Use this expression to calculate the cost of playing table tennis for 3 hours.

Question 2

The cost of a badminton racket is \in 16 and the cost of a shuttlecock is \in 2. The cost (C) of x rackets and y shuttles can be expressed by C = 16x + 2y. Determine the cost when a person buys 6 rackets and 8 shuttles.

Question 3

In soccer, teams are awarded 3 points for games won (w) and one point for games drawn (d). The total points (P) gained by one team can be expressed by P = 3w + d. Use this expression to calculate the total points for Liverpool in the 2008 season in which they won 25 games and drew 11.

Question 4

If a = -1 and b = 2 and c = 4 evaluate:

i.

$$4c - 3b =$$
 10

 ii.
 $3b + 2c - b =$
 12

 iii.
 $-2(a) - b =$
 0

 iv.
 $a + b + c =$
 5

 v.
 $-a - b - c =$
 -5

86

€8.50

€112

1)	x + 7 = 16	9
2)	x + 10 = 15	5
3)	5 + x = 12	7
4)	x - 4 = 8	12
5)	x - 2 = 9	11
6)	x + 7 = 8	15
7)	x - 1 = 14	15
8)	x + 565 = 720	155
9)	-432 + x = 123	555
10)	x - 226 = 624	850

1)	x + 3x = 16	4
2)	2x + 10 = 18	4
3)	5x + x = 12	2
4)	2x - 4 = 8	6
5)	3x - 14 = 7	7
6)	5x - 7 = 8	3
7)	4x - 14 = 2x	7
8)	20 = x + x	10
9)	45 - 2x = 3x	9

10) -2x + 36 = 4x

6

1)	$\begin{aligned} x + y &= 9\\ x - y &= 3 \end{aligned}$	<i>x</i> ≤ 3
2)	2x + y = 8 $3x - y = 2$	<i>x</i> ≤ 5
3)	x + 2y = 7 $2x + y = 8$	$x \le 4$
4)	4x - y = -9 $2x - 3y - 7$	$x \le 2$
5)	2x + y = -2 $x + 3y = 9$	$x \le -2$
6)	3x + 4y = 5 $2x - 3y = 9$	$x \ge -3$
7)	3x + 5y = 6 $2x + 3y = 5$	$x \le 4$
8)	2x + 3y = 3 $x - 4y = 7$	$x \ge -2$
9)	3x - 2y = 13 $4x + 3y = 6$	$x \le 2$
10)	2x - 5y = 3 $x = 3y + 1$	$x \ge -1$

1)	$x + 2 \le 5$	x =6, y = 3
2)	$x-1 \le 4$	x =2,y = 4
3)	$2x - 3 \le 5$	x =3, y = 2
4)	$4x - 5 \ge 3$	x =-2, y =1
5)	$2x + 5 \le 1$	x =-3, y =4
6)	$3x + 4 \ge -5$	x =3, = -1
7)	$3x-5 \le 7$	x =7, y =-1
8)	$4x + 3 \ge -5$	x =3, y = -1
9)	$6 - x \ge 4$	x =3, y =-2
10)	$3 - x \le 4$	X = 4, y = 1

• When 14 is subtracted from President Mary McAleese's age, the result is 44. How old is the President?

Resources

• When 13 is added to Ireland's famous cyclist Sean Kelly's age (x), the result is 66. How old is Sean?

• When 18 is added to the number of red stripes on the American flag, the result is 31. How many red stripes are on the flag?

• When 16 is subtracted from Venus Williams age (x), the result is 13. How old is Venus?

• When 12 is added to Sonia O'Sullivan's age (x), the result is 52. How old is Sonia?

• When 13 is added to Jedward's age (x), the result is 31. How old are the twins?

53



13



18



Dan Shanahan is 11 years older than Joe Canning. If the sum of their ages is 53, find the age of each.

Resources

Grainne Seoige is 4 years older than her sister Sile. If the sum of their ages is 66, find the age of each.

Roy Keane is 7 years older than Brian O'Driscoll. If the sum of their ages is 69, find the age of each.

- Pat Kenny is 25 years older than his successor on The Late Late Show, Ryan Rubridy. If the sum of their ages is 97, find the age of each.
- Padraig Harrington is 18 years older than Rory McElroy. If the sum of their ages is 60, find the age of each.

Henry Shefflin has 4 All Ireland Hurling medals more than Sean Og O hAilpin. Together they have 10. How many has each?

> Sean Og = 3 Henry = 7

Roy = 38

Ryan=36

Pat = 61

Rory = 18

Padraig = 39

Brian = 31

Dan = 32

Joe = 21

Sile = 31

Grainne = 35

• The length of a soccer pitch is 35m more than its width and its perimeter is 330m. Find the width of a pitch.

65m

• The length of a rugby pitch is 30m more than its width and its perimeter is 340m. Find the width of a pitch.

70m

• The length of a hockey pitch is 35m more than its width and its perimeter is 290m. Find the width of a pitch.

55m



Level 4: Mathematics

Unit 4: Data Handling

Identify the (i) population (ii) the sample (iii) the variable measured and (iv) the statistic provided in the following examples:

 A protester wants to know how many people in her local community are interested in travelling with her to Dublin for protest. She sends out letters to 300 households and receives 200 replies. 75 out of the 200 people who replied would be interested in making the journey to Dublin to protest.

Population: All people that live in the protester's local community.

Sample: 200 households that replied to the letter.

Variable Measured: Are you interested in travelling to Dublin for a protest? Yes/No

Statistic: 37.5% (75 out of 200) of people who replied would be interested in making in the journey to Dublin to protest.

 A coffee franchise wish to open on a university campus. An email is sent to all university students and staff to ask them if they would buy coffee from this new shop should it open on campus. 3,400 students replied to the email. 3,000 of those that replied said they would buy their coffee from the shop.

Population: All university students and staff.

Sample: 3,400 staff and students who replied to the email.

Variable Measured: Would you buy your coffee from this new franchise if a shop was opened on campus? Yes/No.

Statistic: 88.2% of students and staff at the university (3,000 out of 3,400) who replied to the email said they would but coffee from the shop if it opened on campus.

135

A primary school class want to measure the growth rate of their plants in school. They
decide to put half the plants under sunlight for four hours and the other half under sunlight
for 8 hours. They will them compare growth levels.

Can you identify the independent variable and the dependent variable in this study?

Independent variable: is the amount of sunlight the plants are put under which is varied by the researcher.

Dependent variable: is the growth rate of the plants. This is dependent on the amount of sunlight the plants received.

 Two classes were given a short training course on speech and language. The speech and language tutor wanted to know if an incentive in the form of money would influence the students' performance in the test taken at the end of the course. The students in one class were offered €5 for getting a certain amount of marks on the test while the other class was not offered any money.

Can you identify the independent variable and the dependent variable in this study?

Independent variable: is the monetary incentive (€5 or no money) offered to students for doing well in the exam.

Dependent variable: is the performance in the test by the students.

• A number of parents were asked how long their children play video games for per week. These were the results (in hours):

1	2	2.5
3	3.75	3
1.5	2	4.5
4	3.20	4
1	0.5	2

15 parents were in the survey.

The data is continuous as it is a measurement that can include decimal points. That is, children can play video games for values other than whole numbers.

 The speed cameras installed on a motorway recorded the speed, to the nearest kilometre, that a number of cars were travelling at over the course of an hour. These were the results (in kilometres):

There were **12 cars** in the survey

Is the data discrete or continuous? Why?

The data is discrete because it records the speed the cars were travelling at to the nearest kilometre which can only consist of whole numbers.

• A number of restaurants were asked what their most popular dish was. The results were as follows:

Lasagne	Vol au vents	Steak
Steak	Chicken Curry	Lasagne
Lasagne	Vol au vents	Vegetarian Stir Fry
Beef Burger	Beef Burger	Chicken Curry
Steak	Lasaone	Beef Burger

Complete the following frequency table:

Favourite	Lasagne	Vol au	Steak	Chicken	Beef	Vegetarian
Dishes		vents		Curry	Burger	Stir Fry
Number of	4	2	3	2	3	1
Restaurants						

15 restaurants were involved in the study.



What was the most popular dish? Lasagne

What was the dish that was mentioned least by restaurants?

Vegetarian Stir Fry

• A survey was carried out which asked a number of teenagers what their preferred social network site is. The results were as follows:

Facebook	Twitter
Facebook	Bebo
	Facebook
Bebo	Facebook
	Facebook
Twitter	MySpace
	Bebo
Twitter	Facebook
	Facebook
Facebook	Twitter
	Facebook

Complete the frequency table representing this data:

Social	Facebook	Twitter	Bebo	MySpace
Network Site				
Number of	9	4	4	1
Teenagers				

18 teenagers took part in the study.

Represent this information on a bar chart (use graph paper).



Can you make any conclusions about Facebook?

Facebook is the most popular social network site among these teenagers. 50% of the teenagers chose Facebook as their favourite social network site.

What was the least popular social network site?

MySpace is the least popular social network site. It was only chosen by one teenager out of the 18 that took part in the study.

• A number of adults were asked whether or not they liked or disliked maths. There was also an option to say they were unsure. Based on the information received, the following pie chart was drawn.



Using the pie chart above, answer the following questions:

How many adults took part in the study?

20 adults took part in the study (add the count number given on each sector of the pie chart i.e. 12+5+3)

What sector was the greatest?

The 'like maths' sector was the greatest sector. It had an angle of 216° and took up 60% of the pie chart (216 divided by 360 or 12 divided by 20 multiplied by 100%).

Why was the angle for "unsure" smaller than the other two angles?

This angle was smallest because the least number of adults in the survey (3 adults out of 20) chose this option.

• The unfinished pie chart below represents the males that took part in a study and the females that took part in a study.



The angle of 144° represents the males.

What angle represents the females?

 216° is the angle that represents the females since $360^{\circ} - 144^{\circ} = 216^{\circ}$.

 A number of couples were asked what their preferred type of accommodation would be. They were asked to choose between a bungalow, a two-storey house and an apartment. The results were put into a frequency table.

Accommodation	Bungalow	2-Story House	Apartment
No. of Couples	3	7	2

In order to complete the following table firstly we need to know how many degrees one couple will represent on the pie chart.

12 couples took part in the survey. If 2 couples = 360° then 1 couple = 30°

(360° divided by 12 to get the value for 1 couple)

So one couple takes up 30° on the pie chart.

Accommodation	Number of Couples	Angle
Bungalow	3	$3 \times 30^{\circ} = 90^{\circ}$
2-Story House	7	7 x 30º = 210º
Apartment	2	2 x 30 = 60°
Total	12	360°

Construct a pie chart based on this information: see next page.


Answers:

12 couples took part in the survey.

210° was the greatest angle.

This angle represents the 7 couples whose preferred choice of accommodation is a 2-storey house.

Conclusion: The majority of the couples said they would like to live in a 2-storey house (58.3% or 7 out of the 12 couples). There was only one couple in the difference between the bungalow and an apartment as a preferred type of accommodation. 25% of the couples said they would like to live in a bungalow (3 out the 12 couples) while 16.7% (2 out of the 12 couples) said they would like to live in an apartment.

A 4th class tutor decides to graph her students' attendance over the course of five weeks.
 There are 25 students in her class.



How many students were present in week 1? 24

How many students were present in week 2? 25

How many students were present in week 3? 20

How many students were present in week 4? 19

How many students were present in week 5? 23

Was there any time when the class was at full attendance? Yes. In week 2 there was full attendance (25 students).

Can you summarise the trend in the students' attendance?

There was only one student absent in week 1 with full attendance in week 2. There was a decline in attendance for the next two weeks with an incline to 23 students in the final week.

 A shop that sells ice-cream is trying to decide whether it should have ice cream on sale all year round or just in the summer months. The shop decides to look at last years ice cream sales to help make their decision.

They firstly put the results into a frequency table:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
No. of	10	15	38	45	60	100	130	120	90	20	10	5
Galoo												



Trend graph:

Answers:

July had the highest ice cream sales.

December had the lowest ice cream sales.

Do you recommend any months were the shop should consider not selling ice cream? Why /Why not?

Sales were lowest in the months of November, December, January and February and possible the shop should consider not selling ice cream in those months. The shop should look at the cost of producing and buying the ice cream versus the money made on the sale of a low number of ice creams over those four months.

A number of women were asked how many times they get their hair cut a year. The results are represented on the following histogram.



Answer the following questions based on the histogram:

- (a). How many women got their hair cut 0-2 times a year?2 women
- (b). How many women got their hair cut 2-4 times a year?10 women

(c) How many women got their hair cut 4-8 times a year?

20 women got their hair cut 4-8 times a year since this unit is twice the size of the smallest units (0-2 for example) and the height of 10 must be multiplied by 2 to get the correct number of women who got their hair cut 4-8 times a year.

Resources

- (d). How many women got their hair cut 8-12 times a year?
 16 women got their hair cut 8-12 times a year since again, the height must be doubled to get the correct frequency (8 x 2).
- (e) Complete the grouped frequency table based on the information you just calculated:

Number of hair	0-2	2-4	4-8	8-12
cuts				
Number of	2	10	20	16
women				

- (f) How many women took part in the survey?48 women took part in the survey.
- (g) Can you draw any conclusions from the data about the number of times women get their hair cut during the year?

4-8 times a year is the most frequent number of times that a woman in this survey gets her hair cut. Almost 42% said 4-8 times (20 out of 48).

Very few in this survey get their hair cut only 0-2 times (approx 4%).

The majority get their hair cut more than 4 times throughout the year. 36 out of the 48 (75%) said they get their hair cut 4-8 times or 8-12 times a year.

1. Based on the following information construct a histogram (use graph paper)

and answer the question that follows.

A local shop wants to know the amount spent by customers in their shop on a given Saturday. They collect the following data.

Amount (€)	0-6	6-12	12-24	24-36	36-42
Number of	5	12	10	8	2
customers					



What **conclusion**, if any, can the shop make about the amount of money spent in the shop on that Saturday?

Only 5 customers spent the least amount of money (\in 5). The most frequent amount of money spent in the shop that Saturday was \in 6- \in 12. After this, as the amount of money spent increased, the number of customers spending such amounts decreased. This lack of large spending might suggest that the shop is not a large supermarket where a weekly shop is done by customers. Perhaps it is more of a medium sized shop.

2. Based on the following information construct a grouped frequency table

and a histogram (use graph paper).

The numbers in the box below represent the number of local GAA matches attended by a number of fans

1	10	3	14	2	3	
17	4	9	2	8	5	
20	3	21	7	1	26	

Complete the grouped frequency table:

GAA matches	0-4	4-8	8-16	16-24	24-28
Number of fans	8	3	3	3	1

Represent this information on a histogram:



1. Complete the following cumulative frequency table based on the grouped frequency table.

Time (minutes)	0-20	20-40	40-80	80-100	100-120
No. of teenagers	12	15	8	6	1

Cumulative frequency table:

Time (in minutes)	<0	<20	<40	<80	<100	<120
No. of teenagers	0	12	27	35	41	42

2. Draw a cumulative frequency curve based on the completed cumulative frequency table.



Δ		
$\Box v$	C I	

Work out the mean, median and mode for each of the following sets of numbers:

i.	7	9	8	9	3		
Меа	an:						
	36 divided by $5 = 7.2$						
		7.2 is the	mean num	ber of thes	se numbers	5.	
Мес	dian:	8					
		Organise o	data in ord	er firstly: 3	8, 7, 8, 9, 9	Ð	
		8 is the m	iddle numb	ber			
Мо	de:	9 (occurs	twice)				
ii.	1	4	7	11	13	8	1
Меа	an:	1+4+7+1	1+13+8+1	. = 45			
		45 divided	l by 7 = 6.4	4			
		6.4 is the	mean num	ber of thes	se numbers	5.	
Мес	dian:	7					
		Organise o	data in ord	er firstly: 1	., 1, 4, 7, 8	3, 11, 13	
		7 is the m	iddle numb	ber			
Мо	de:	1 (occurs	twice)				

Level 4			I	Mathematics				
iii.	0.2	5.2	4.1	0.2	6.5			
	Mean:	0.2 + 5.	0.2 + 5.2 + 4.1 + 0.2 + 6.5 = 16.2					
		16.2 div	vided by 5 =	3.2				
		3.2 is th	ne mean nun	nber of these	e numbers.			
	Median:	4.1						
		Organis	se data in oro	der firstly: 0.2	2, 0.2, 4.1, 5.2,	, 6.5		
		4.1 is the middle number						
	Mode:	0.2 (oc	curs twice)					

iv. The figures below represent the number of phone calls received by the Garda that report house burglaries over seven days.

Number of phone calls	Number of days
1	3
2	3
3	4
4	2
5	1

What is the mean number of phone calls the Garda received?

Answer: 3

Calculate all the values: $(1 \times 3) + (2 \times 3) + (3 \times 4) + (4 \times 2) + (5) = 34$. This number is then divided by the number of phone calls which is 13. 34 divided by 13 is 2.6. The mean number of phone calls that the Garda received over the 7 days was 2.6.

What is the median number of phone calls the Garda received?

Answer: 3

Arrange the data in order:

1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5

The middle number is 3. That is, the median number of phone calls received by the Garda is 3.

What is the mode?

Answer: 3 is also the mode because it occurs most often (4 times).

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