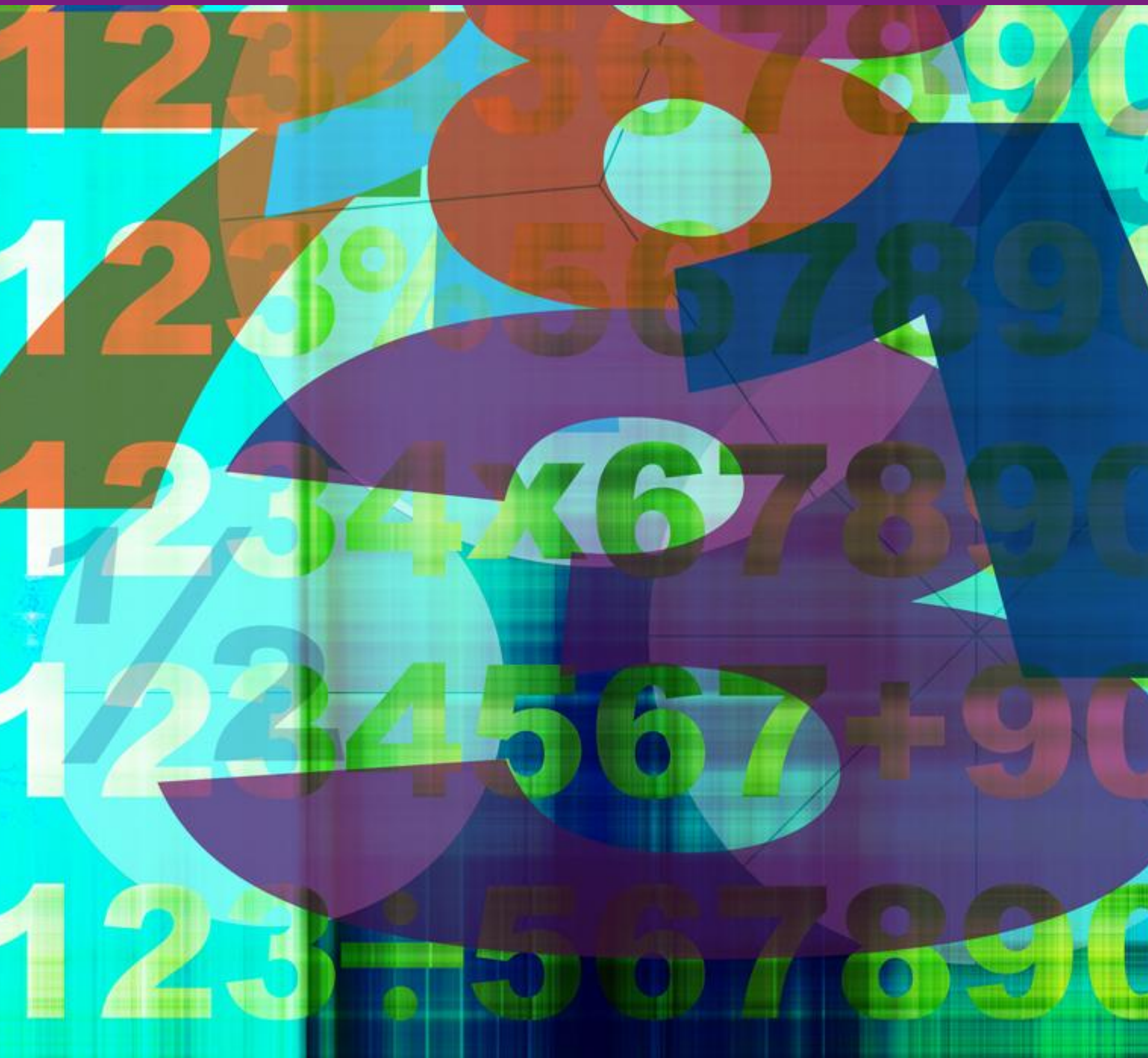
Learner Pack

Level 4: Mathematics
Unit 2: Geometry



Activity **Soccer, Snooker and Handball** **Code G1**

This activity links to award learning outcome 2.1.

Introduction

Geometric shapes form part of our everyday experience. They are present in our homes, our work and our sports.

Learning Outcomes

1. Recognise circles and rectangles.
2. Recognise rectangles and triangles.
3. Recognise rectangles and squares.

Key Learning Points

1. Geometric Shapes

What do you need to know before you start?

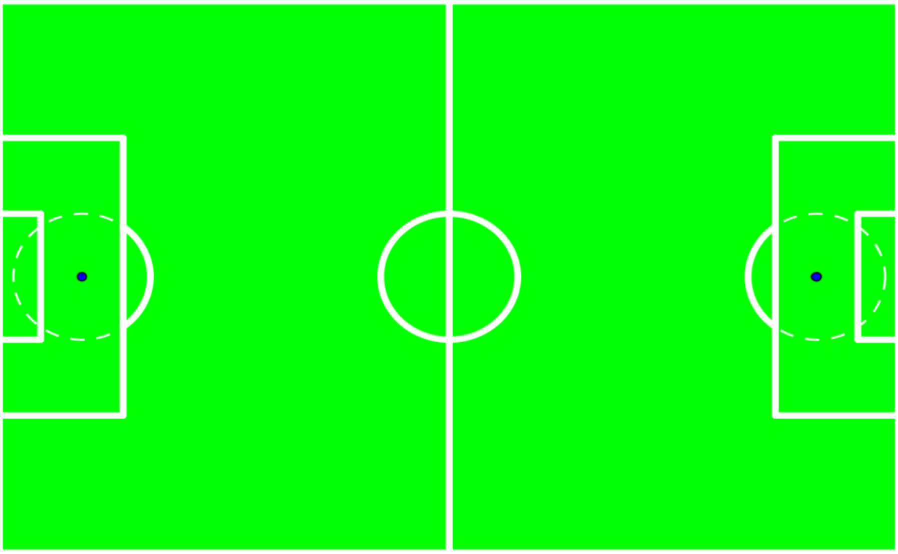
Name	Shape
Circle	
Rectangle	
Triangle	
Square	

* You also need to know the possible dimensions of the sports' playing surfaces.
* You need to know the size and shape of the various areas in the playing surface.

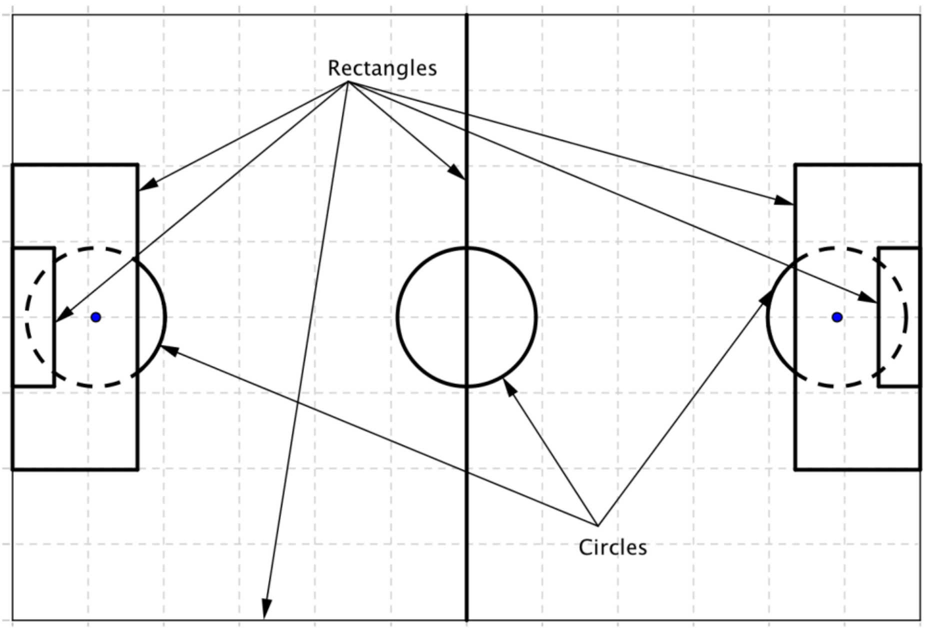
Task 1: Soccer

Example

Mark the rectangles and circles you can see in this drawing of a soccer pitch.

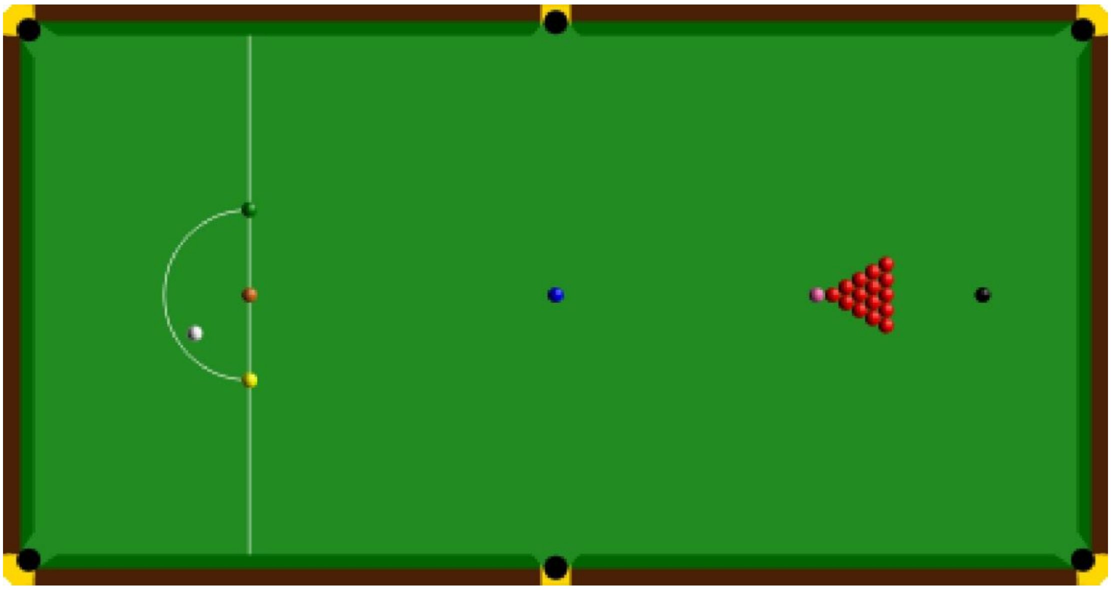


Solution



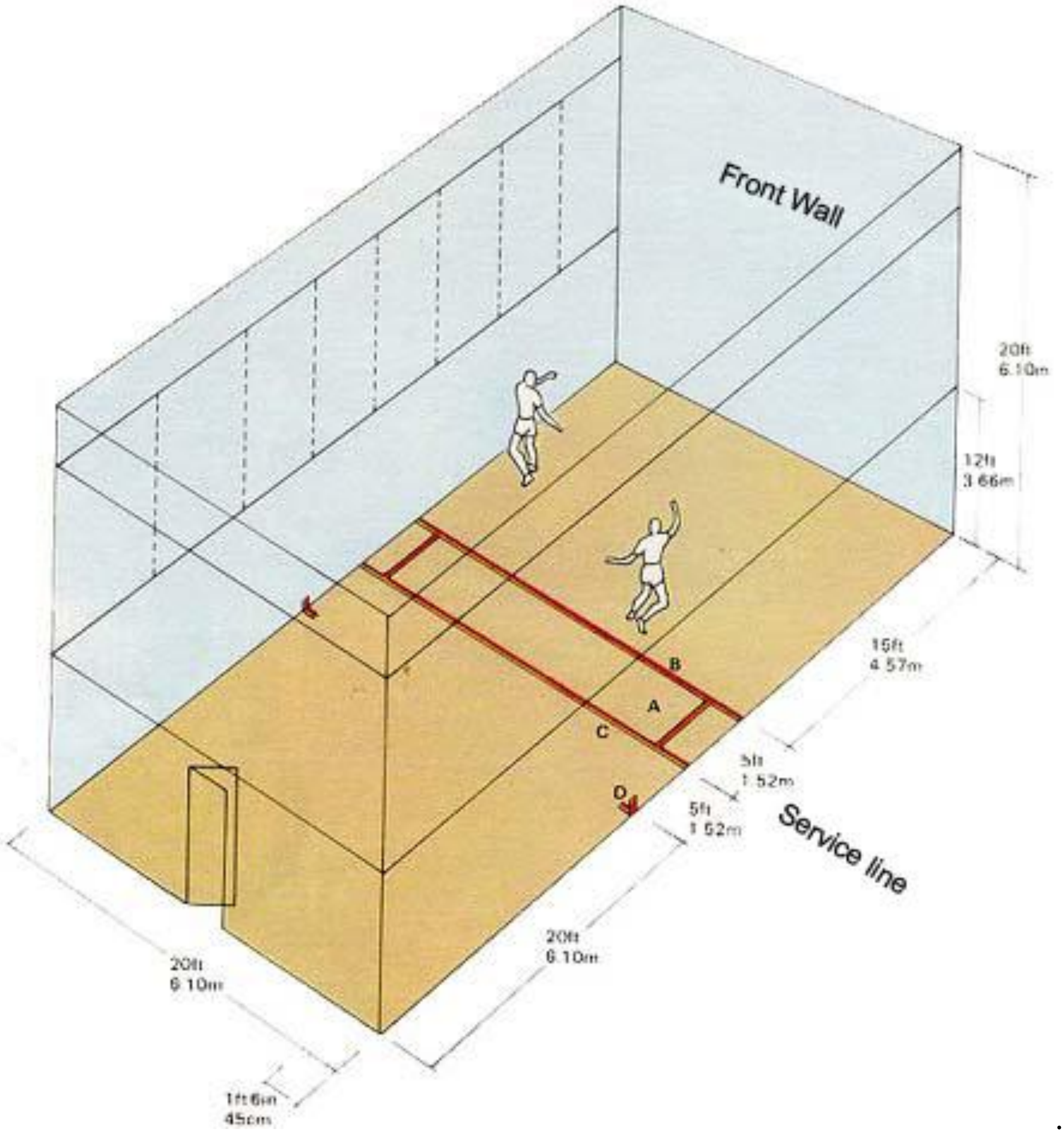
Now you try this

- Below is the diagram of a snooker table. Mark the rectangles and triangles you see in it. Can you also mark a semicircle – that is, a figure which is half of a circle?



Now try this

- This is a diagram of a handball court. Mark the square that appears there.



Practise your skills

- Practice sheet G1

Activity **Mirror, mirror on the wall…** **Code G2**

This activity links to award learning outcome 2.2.

Introduction

This activity will help you understand the basic ideas of symmetry.

Materials you will need

- Pen & Paper
- A mirror
- Internet

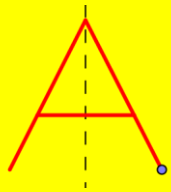
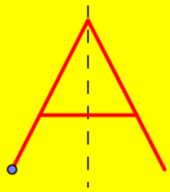
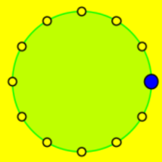
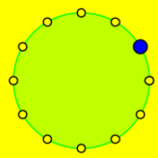
Learning Outcomes

1. Recognise when an object can be folded onto itself.
2. Recognise when an object can be turned around onto itself.

Key Learning Points

1. Folding symmetry
2. Rotational symmetry

What do you need to know before you start?

On the right there are two drawings of the letter “A”. The one on the right is the one on the left folded over the dotted line. If the small dot had not been included you would not know that they were different. Figures that can be folded over to go on top of themselves like this have folding symmetry.		
On the right there are two drawings of a dinner plate. The colour of one of the decorative spots has been changed to blue. If this had not been done you would not know that they were different. Figures that can be turned through some angle to go on top of themselves like this have rotational symmetry.		

- If a letter has vertical folding symmetry it will not seem to change when looked at in a mirror.
- If an object has rotational symmetry it can be turned through some angle to coincide with itself.

Task 1:
Messages in a mirror

Example

If we hold letters up to a mirror, some of the letters look unchanged in the mirror and some look changed. We read from left to right, but in a mirror the direction of the sentence will usually be reversed. However, if we write the sentence vertically – putting the words from top to bottom instead of left to right - it will appear unchanged in a mirror **if** all the letters have folding symmetry.

Write the sentence **I AM TOM** in capital letters. Will it look unchanged in a mirror if you print it vertically?

Solution

Hold this sheet of paper up to a mirror. See that the sentence "I AM TOM" looks the same in the mirror**.**

I

A
M

T
O
M

Now you try this

There are five phrases below. Some of them will appear unchanged in a mirror if written vertically. Some will not. For each phrase, say whether or not it will be unchanged. If it will be changed say what letter is causing the change.

Write the phrases vertically on paper and use a mirror to check your answers.

- I AM TONY
- I HIT TIM
- THAT HAT ON YOU
- WE HIT TOM
- THAT HAT WITH YOU

- Think of some short phrases that will be unchanged when you see them in a mirror.

Looking for rotational symmetry

Many objects created by artists and designers have rotational symmetry.
This is the well known symbol of the Isle of Man.

It can be turned through one third of a full turn to fall on top of itself.

It has threefold symmetry.

Now you try this

Hubcaps on the wheels of cars very often show rotational symmetry. Sometimes the design will incorporate the company's name. This can break the symmetry.

- Use the Internet to find and download images of hubcaps.
- Mark three of these that have rotational symmetry.
- Mark three that do not have rotational symmetry.
- Mark one image that does not have rotational symmetry but does have folding symmetry.

Practise your skills

- Practice sheet G2

Activity **Hanging cups** **Code G3**

This activity links to award learning outcome 2.3

Introduction

Hanging objects on a wall is an activity that most people undertake at some point. Normally we need to plan before doing this. We need to decide exactly where to insert the hooks in the wall to hold cups.

Learning Outcomes

1. Understand the need for coordinates.
2. Plot points in the plane.
3. Read the coordinates of points in the plane.

Key Learning Points

1. Coordination of the plane

What do you need to know?

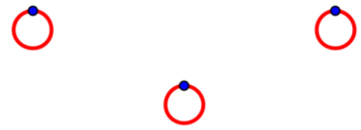
You need to be able to measure distance both in a horizontal and vertical direction. These measurements will help you place the hooks in the correct positions. If you are measuring a distance then you must be measuring from one place to another. The place you are measuring from is called the origin.

You need to be able to draw out a plan on paper and transfer measurements from paper to a wall.

Task 1: Hanging cups

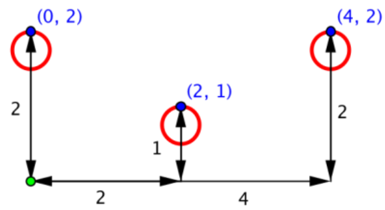
Example

We want to hang cups on a kitchen wall as in this drawing:



Pick a point on the wall to measure from. This might be where the wall meets a worktop. On paper, draw a design that you like and that is practical. When you have this done, measure the distances to the hooks from the point that you picked. Label each point as a pair of numbers. The first number is the **horizontal** distance from the reference point. The second number is the **vertical** distance from this point.

Solution

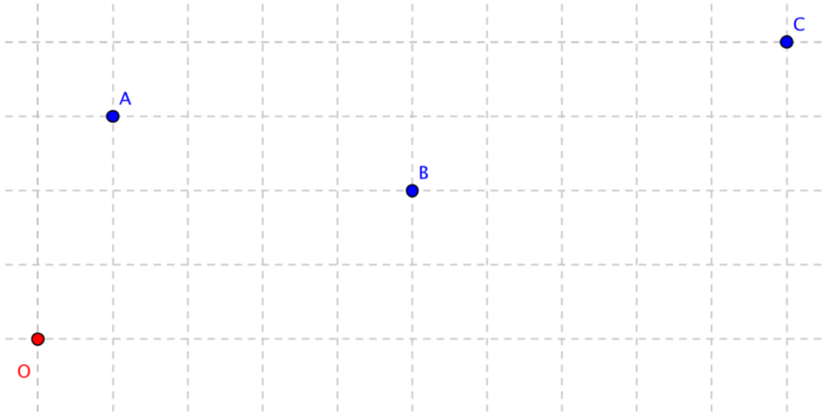


The hooks will be placed at the points (0, 2), (2, 1) and (4, 2).

You can scale up the measurements and transfer to a real wall.

Task 2: Coordinates

- In the diagram below measure the vertical and horizontal distances of the three points *A*, *B*, and *C* from the point labelled *O*. Write these numbers as coordinates.



- Estimate the coordinates on the maps your tutor will give you.

Practise your skills

- Practice sheet G3

Activity **Cookery lessons** **Code G4**

This activity links to award learning outcome 2.3

Introduction

Many more people than ever are cooking at home rather than buying fast food. Many people use cookbooks to help prepare home meals. In many cases the temperature in older recipes is given in Fahrenheit. The temperature of modern ovens is given in Celsius, so we need to be able to convert from Fahrenheit to Celsius and vice versa.

Materials you will need

- Graph paper
- Pen and paper

Learning Outcomes

1. Graph ordered pairs.
2. Read information from a graph.

Key Learning Points

1. Graphing ordered pairs

What do you need to know before you start?

Maths

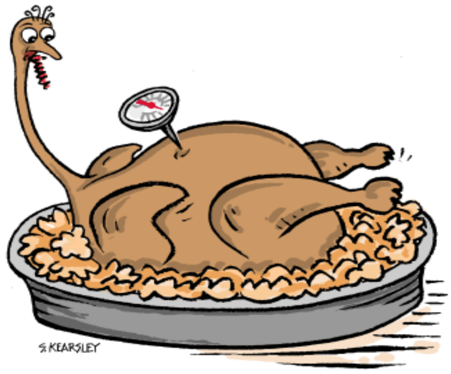
You need to know how to plot points in the coordinate plane and how to read points from a picture of the plane.

Temperature

You need to know how to read temperature in Fahrenheit and Celsius. To convert from Fahrenheit to Celsius this is what you do:

1. Subtract 32
2. Divide by 9
3. Multiply by 5

For example to convert 140 degrees Fahrenheit to Celsius



1. Subtract 32: 140 – 32 = 108
2. Divide by 9: 108/9 = 12
3. Multiply by 5: 12 x 5 = 60

Therefore 140 degrees Fahrenheit is 60 degrees Celsius.

Task 1: Making a graph

Example

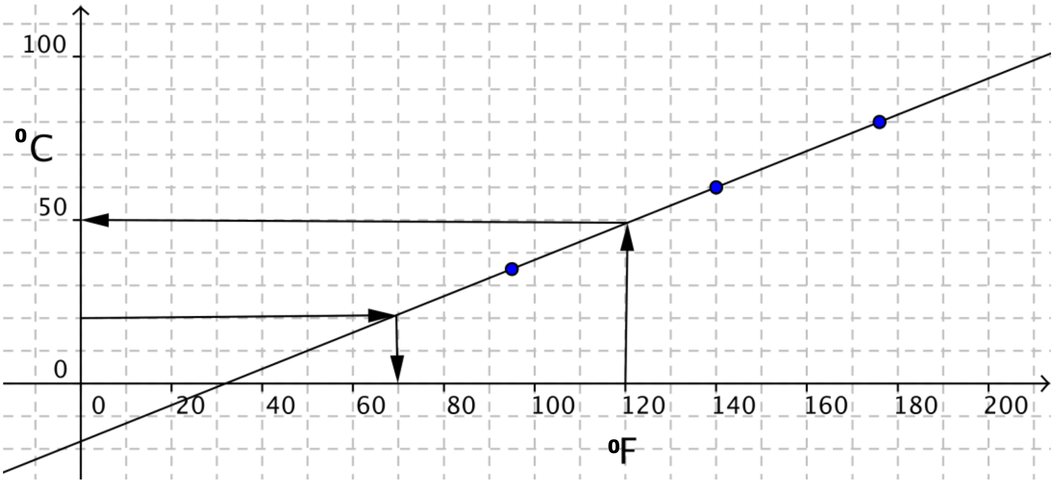
John was preparing food which needed to be cooked at 140 degrees Fahrenheit. He converted this temperature to degrees Celsius and found that this was 60 degrees.

Mary was cooking food for her family and used a recipe that required a temperature of 176 degrees Fahrenheit (80 ^oF).
She calculated that this was the same as 80 degrees Celsius (80 ^oC).

Joan prepared a dish that only required a temperature of 95 ^oF. She had calculated that this was the same as 35 ^oC.

Graph the ordered pairs above with Fahrenheit on the x-axis and Celsius on the y-axis.
Use your graph to estimate 120 ^oF in Celsius and 20 ^oC in Fahrenheit.

Solution



Now you try this

- A car is starting from its parking place at the road side. The driver accelerates smoothly to give the following values of speed and time:

Time (s)	3	5	8	10	12
Speed (m/s)	1.2	2.0	3.2	4.0	4.8

- **Graph these values putting time on the x-axis and speed on the y-axis.**
- **What is the car's speed after 6 seconds?**
- **What is the time it takes the car to reach a speed of 4.4 m/s?**

Practise your skills

- Practice sheet G4

Activity **As the Crow Flies…** **Code G5**

This activity links to award learning outcomes 2.4, 2.5 and 2.8

Introduction

This activity will help you to practise calculating distance and using the Pythagoras Theorem.

Materials you will need

- Ruler, compass, protractor
- Pen and paper

Learning Outcomes

1. Calculate the distance between two points.
2. Know and use Pythagoras Theorem.

Key Learning Points

1. Pythagoras Theorem and use of the Pythagorean formula

What do you need to know before you start?

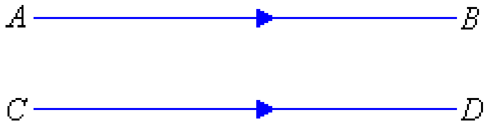
You need to know how to measure distance and how to get squares and square roots on a calculator.

You need to know your lines and angles, and the Pythagoras Theorem.

Know your lines and angles.

Parallel Lines

If two lines do not intersect, then the lines are said to be **parallel**.

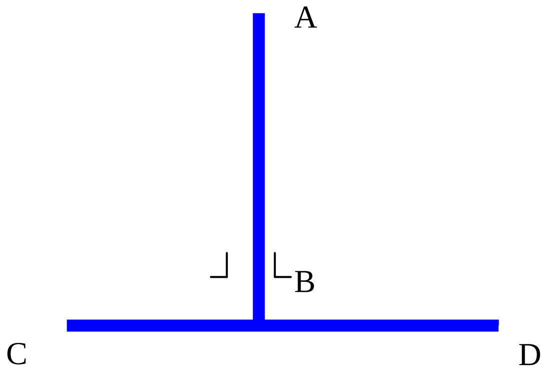


For example, AB is parallel to CD and we write it as $AB \parallel CD$.

Arrows are placed on the lines *AB* and *CD* to indicate that they are parallel.

Perpendicular lines: 'At right angles'

Perpendicular means "at right angles". A line meeting another at a right angle, or 90°, is said to be **perpendicular** to it.



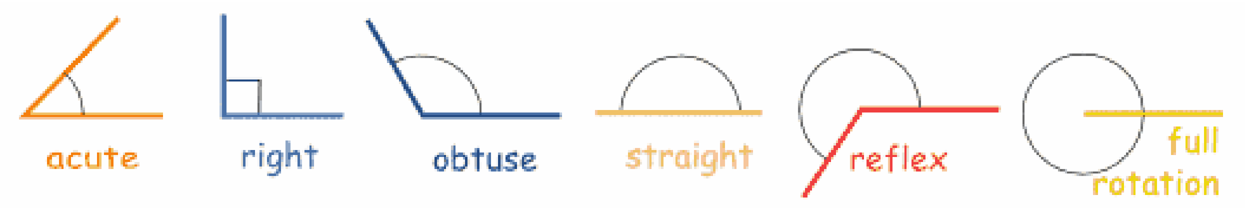
The line AB is perpendicular to CD and we write it as AB $\perp$ CD

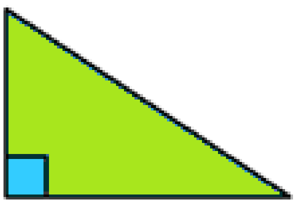
Angles

Angles are measured in degrees (°)

As the angle increases, the name of the angle changes

Type of Angle	Description
Acute Angle	an angle that is less than 90°
Right Angle	an angle that is 90° exactly
Obtuse Angle	an angle that is greater than 90° but less than 180°
Straight Angle	an angle that is 180° exactly
Reflex Angle	an angle that is greater than 180
Full Rotation	A full rotation around the centre of a circle is 360°





This triangle is a right angled triangle. The longest side, opposite the right angle, is called the **hypotenuse.**

Knowing lines and angles

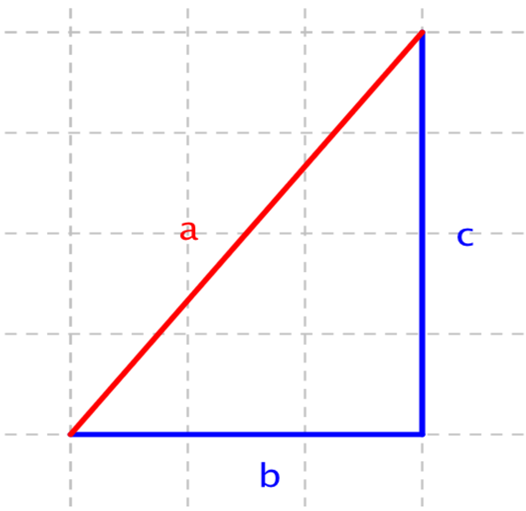
- Look around the room. Notice lines and angles in what you see: for example, in the furniture, the floor, the windows, the open door, the pictures, the hands of the clock, the light hanging from the ceiling, the papers, pens, phones, books on the table and how they are arranged together. Can you see at least one example of the different types of angles mentioned above?

- Construct (draw) a variety of shapes according to criteria your tutor will give you. In each case, label each of the **angles**, using its correct name to show what type of angle it is.

- Draw a right angled triangle according to the measurements your tutor will give you. Label the side opposite the right angle, using its correct name.

Introducing the Pythagoras Theorem

The theorem of Pythagoras says that in a **right angled triangle** the square on the hypotenuse is equal to the sum of the squares on the other two sides of the triangle.



Try the task on the next page: it will help you understand what this means.

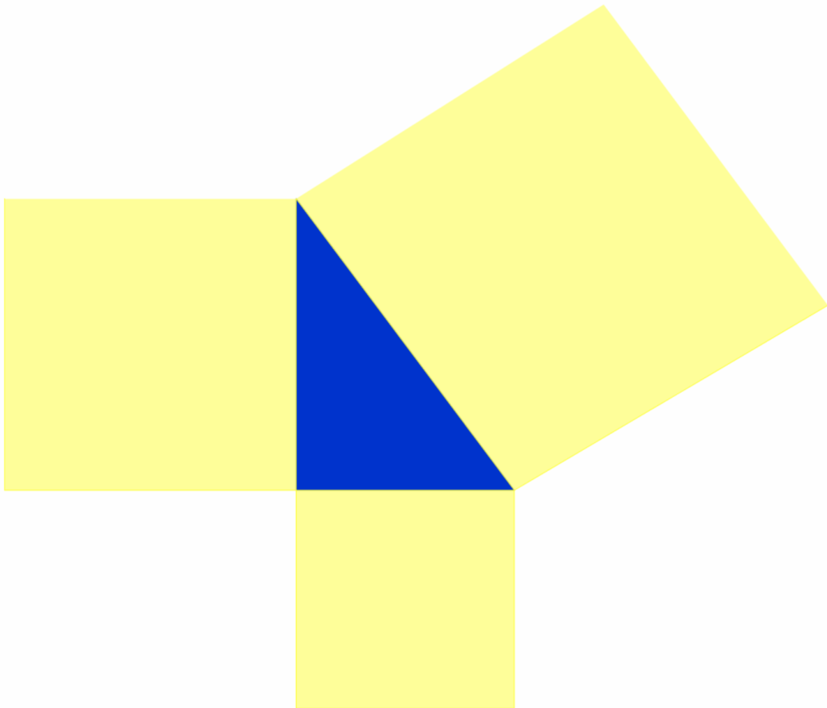
$$a^2 = b^2 + c^2$$

Task 1: Introducing Pythagoras

Example

Look at the diagram below. Imagine all the yellow squares are real gold and you have been asked to choose as follows:

Would you choose the big gold bar or both of the smaller gold bars together?



Which would you choose? ______________________________

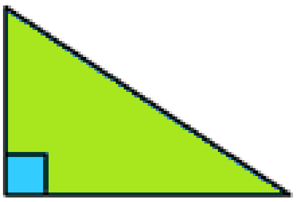
Why would you choose that? ____

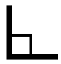
____

Solution

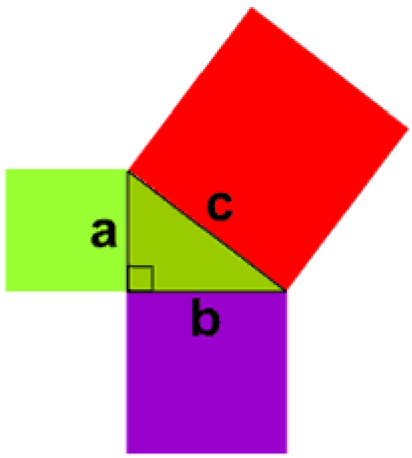
Believe it or not, the big gold bar would have the same amount of gold as the two small bars together. Hundreds of years ago, a man named **Pythagoras** proved that amazing fact about right angled triangles.

Remember:-A right angled triangle is a triangle which has a right angle (90°) in it, like this one:

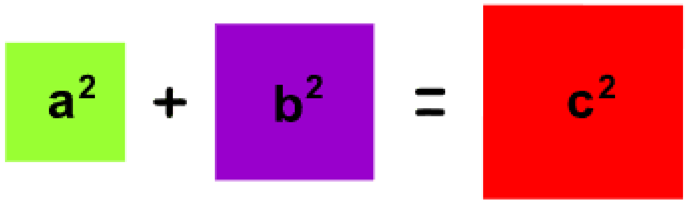


In the Maths world the little square in the corner of the 90° angle  shows us that the angle is a right angle.

Look at the diagram below. It shows a **square** on each of the three sides of this right angled triangle. Pythagoras proved that the biggest square has the same area as the area of the other two squares added together!



This is called "**Pythagoras' Theorem**" and it can be written in one short equation:
$\mathbf{a^2 + b^2 = c^2}$



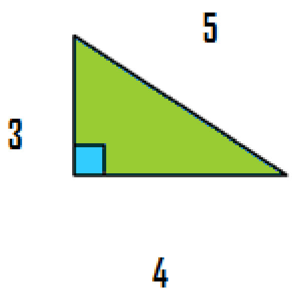
Maths language

Look again at the diagram on the previous page. The triangle's sides are labelled a, b and c. Side c is the longest side of the triangle. In the maths world it is called the **hypotenuse**.

In the maths world the definition of the Pythagorean theorem is:

In a right angled triangle
the square of the hypotenuse is equal to
the sum of the squares of the other two sides.

Check it out



This triangle is a right angled triangle and it has three sides of different lengths. The longest side is called the **hypotenuse.** In this right angled triangle the hypotenuse is 5 units long. The other two sides are 3 units and 4 units long. According to the Pythagorean Theorem the square on the longest side (5^2) should be equal to the sum of the square of the other two sides (3^2 and 4^2).

$\mathbf{5^2 = 3^2 + 4^2}$
$\mathbf{25 = 9 + 16}$
$\mathbf{25 = 25}$

In the maths world 3, 4, 5 in this example is called a **Pythagorean triplet.**

You can use Pythagoras's Theorem to find the length of any side of a right angled triangle if you know the length of the other 2 sides.

Now you try

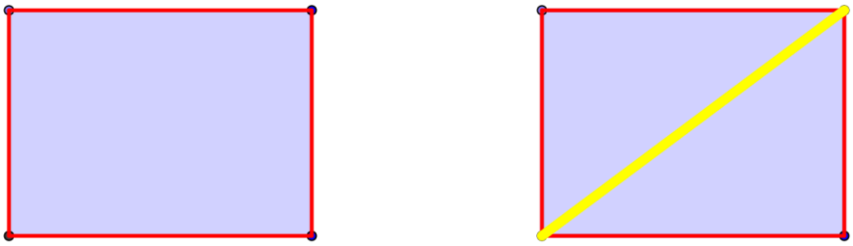
See the previous page, under 'Check it out', for an explanation of what Pythagorean triplet is. Then try this!
Look at the list below. Each line has three numbers. Work out which of these 'triplets' are Pythagorean triplets. Tick 'yes' beside the lines that are Pythagorean triplets.

			Yes	No
5	12	13	☐	☐
7	24	25	☐	☐
9	40	41	☐	☐
13	84	85	☐	☐
12	23	26	☐	☐
9	12	13	☐	☐

Task 2: Finding diagonal length

Example

On the left below is a **drawing of a swimming pool**. The owner wants to add a decorative bridge spanning the pool as in the drawing on the right. To estimate the amount of material needed to build the bridge he needs to know the diagonal length of the pool. The water in the pool prevents a direct measurement. How can he use his knowledge of the width and length to calculate the diagonal distance?



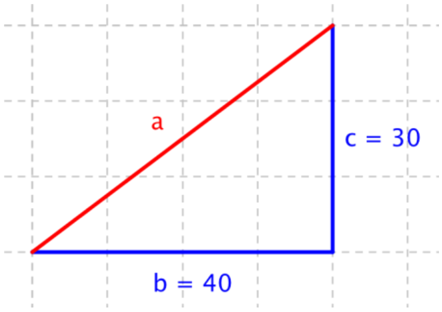
Solution

The owner can use a tape measure to find the width of the pool.

Next he can use a tape measure to find the length of the pool.

The corner of the pool is a right angle so he can draw two sides of the pool as a right angled triangle.

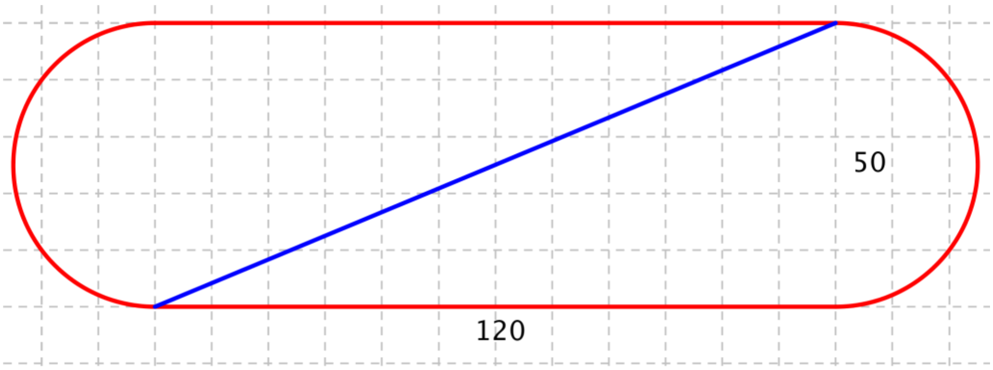
He can then apply the theorem of Pythagoras to this triangle and calculate the diagonal length as follows:



$$
\begin{aligned}
a^2 &= b^2 + c^2 \\
a^2 &= 40^2 + 30^2 \\
a^2 &= 1600 + 900 \\
a^2 &= 2500 \\
a &= \sqrt{2500} \\
a &= 50
\end{aligned}
$$

Now you try this

A racing track is in the shape of a rectangle with semi-circular ends. It is 120m long and 50m wide. Complete the diagram below to show the length and breadth as the sides of a right angled triangle.



- Use the theorem of Pythagoras to calculate the diagonal length, marked by the blue line.

Practise your skills

- Practice sheet G5

Activity **Measuring the world** **Code G6**

This activity links to award learning outcomes 2.4 and 2.5

Introduction

This activity will help you develop skills involved in using maps, including understanding co-ordinates and being able to calculate the distance between points.

Materials you will need

- Graph paper
- Cardboard
- Mathematical instruments

Learning Outcomes

1. Calculate the distance between two points given their coordinates.

Key Learning Points

1. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

What do you need to know before you start?

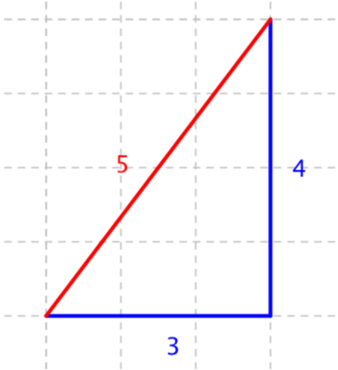
You need to know the theorem of Pythagoras:

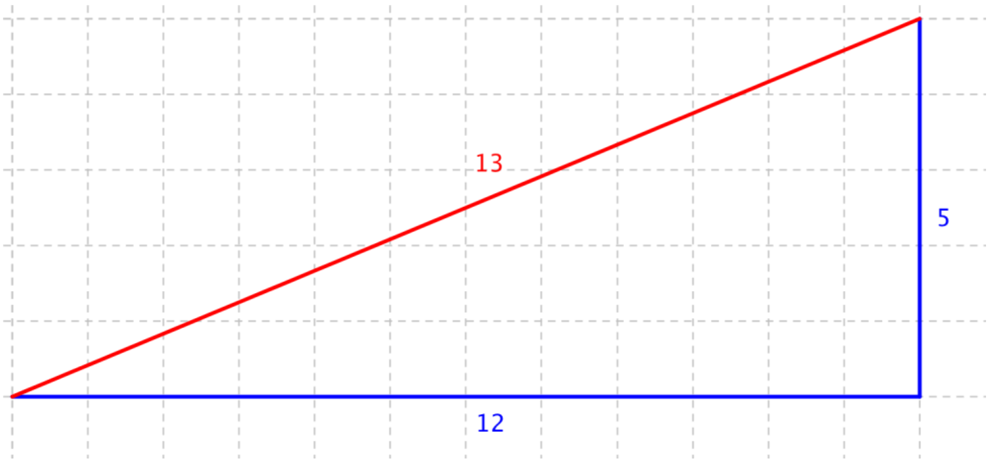
$$a^2 = b^2 + c^2$$

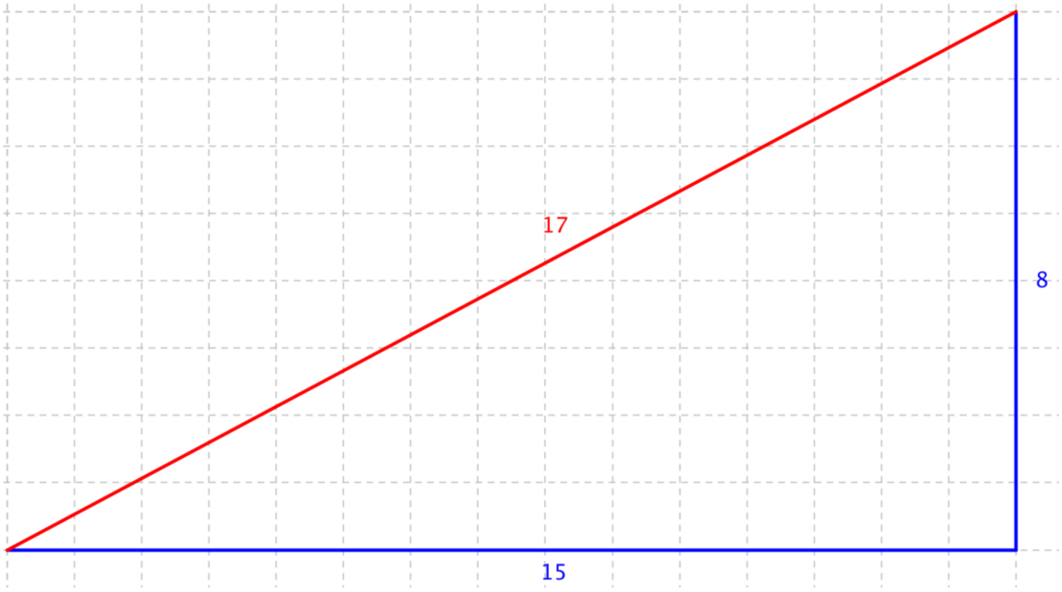
Task 1: Measuring distance with triangles

Example

Construct the three right-angled triangles illustrated below. In each case measure the two vertical sides (in blue). The diagonal will then be fixed automatically to the correct value.







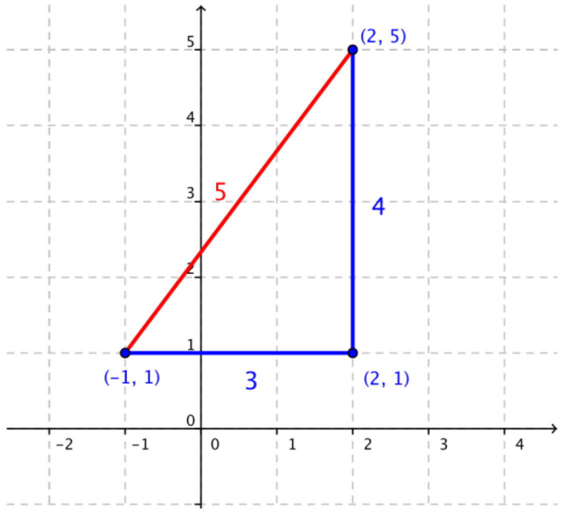
Copy the triangles onto light cardboard and cut out the shapes.

You can **place these triangles on the plane** to measure the distance between various points.

Place the first triangle with the red side joining the points (-1, 1) and (2, 5), as shown below. You will see that the distance between these two points is 5.

Notice that **the base of your triangle** stretches from (-1, 1) to (2, 1) which is three units. We could have calculated this as $x_1 - x_2 = 2 - (-1) = 2 + 1 = 3$.

Notice that **the height of your triangle** stretches from (2, 1) to
(2, 5) which is four units. We could have calculated this as
$y_1 - y_2 = 5 - 1 = 4$



The theorem of Pythagoras tells us that the length of the red line must be

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

You can use the three triangles you have made to measure the distances between various points. However, there are an infinite number of points. **If they do not match up with one of your triangles then the use calculation.**

In the triangles above, you could calculate the length of the red line like this:

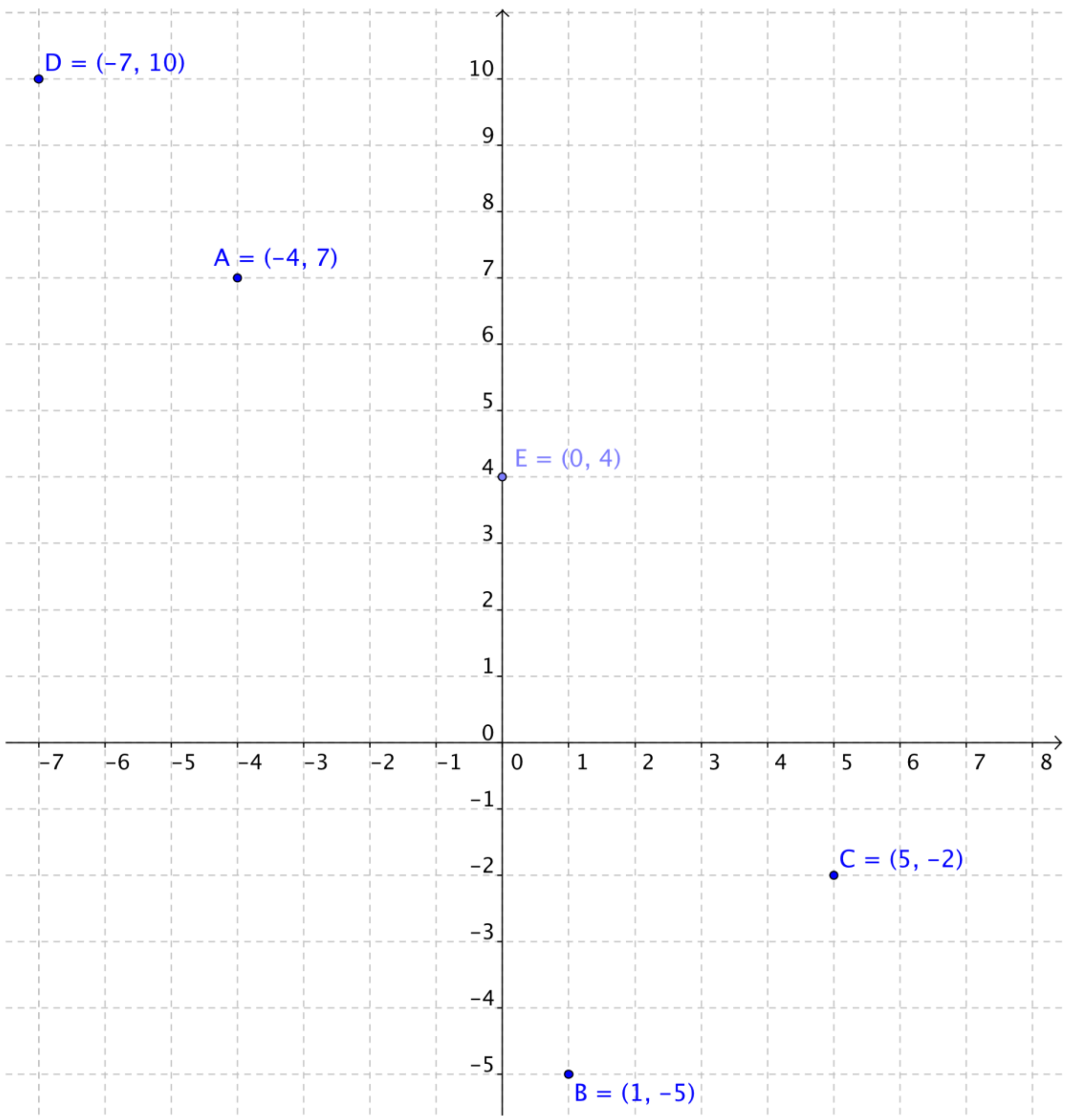
$$
\begin{aligned}
d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
&= \sqrt{(2-(-1))^2 + (5-1)^2} \\
&= \sqrt{(2+1)^2 + (5-1)^2} \\
&= \sqrt{3^2 + 4^2} \\
&= \sqrt{9+16} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

Now you try this

- Use the triangles you have made to find the distances
  AB, BC, BD and AE.

See the next page for more practice in calculating distance between points.

Now you try this



- Calculate the distances between these points:

Points	A and B	Distance	____________
Points	B and C	Distance	____________
Points	B and D	Distance	____________
Points	A and E	Distance	____________

- Calculate the distance between the two points A and D.

Practise your skills

- Practice sheet G6

Activity **Finding the centre** **Code G7**

This activity links to award learning outcome 2.4.

Introduction

This activity will help develop your skills in drawing and reading graphs.

Materials you will need

- Pen and paper

Learning Outcomes

1. Find the mid-point of a line segment.

Key Learning Points

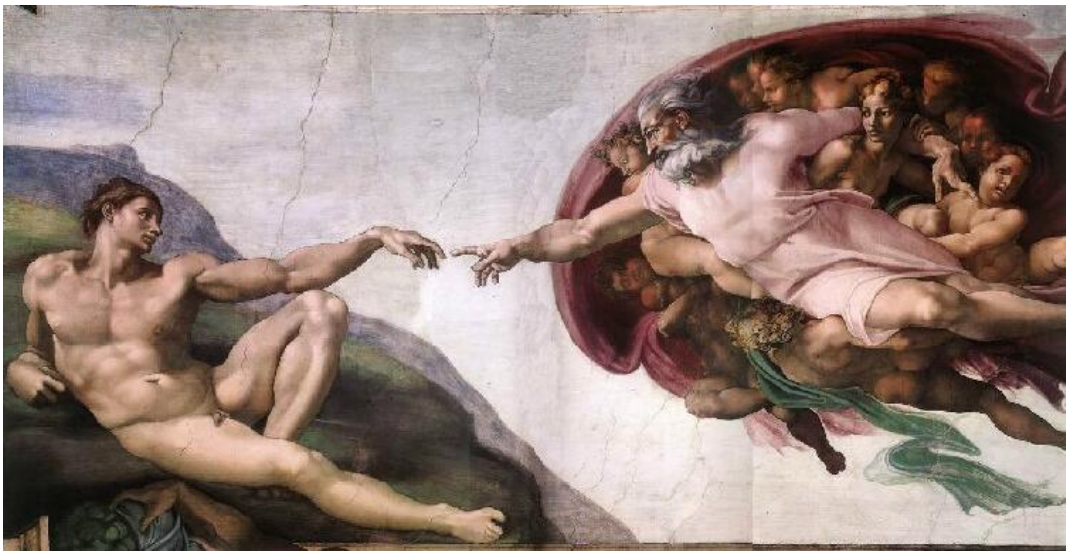
1. $c = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

What do you need to know before you start?

You need to know how to measure length and distance.

Task 1: Finding the centre of a painting

Example



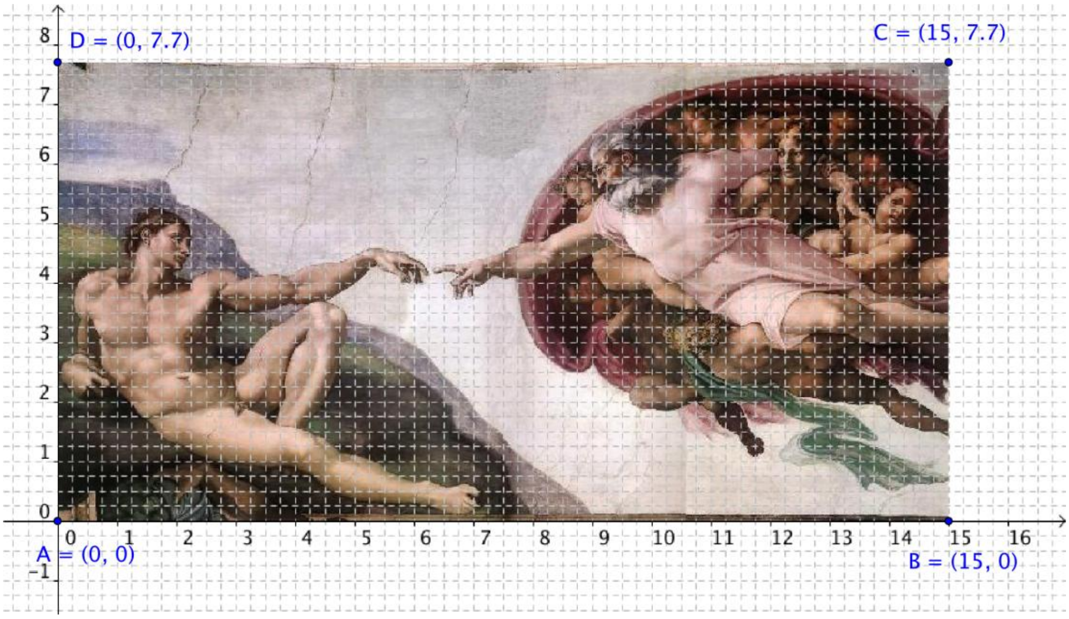
This is part of a famous painting by Michelangelo. It is painted onto the ceiling of the Sistine Chapel in Rome.

In pairs, decide where you think the centre of this image is.

Then work through the calculations on the next page to check your answer.

Solution

We cannot directly measure where the centre of this image is. To do that, we would have to climb up to the ceiling of the Sistine Chapel! However, this is a **photograph of the image superimposed on graph paper: we can use this to measure where the centre is.**



The coordinates of the four vertices are (0,0), (15,0), (15,7.7) and (0,7.7). The centre of the painting must be the mid-point of the lines AC and BD. You can calculate these using A and C as follows:

$$
\begin{aligned}
c &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\
&= \left(\frac{0+15}{2}, \frac{0+7.7}{2}\right) \\
&= \left(\frac{15}{2}, \frac{7.7}{2}\right) \\
&= (7.5, 3.85)
\end{aligned}
$$

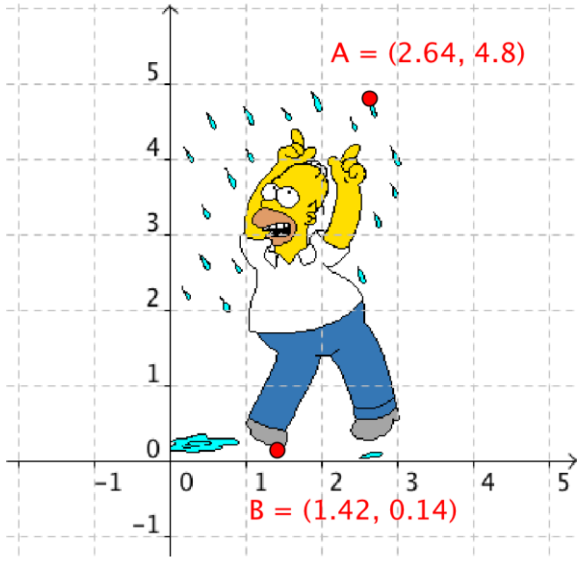
You will get the same answer if, instead, you use B and D. **This point is the centre of each line and is the centre of the painting.**

The centre is not the point where the finger of God almost touches the finger of Adam.

Now you try this

Two points A and B are marked in the drawing of Homer Simpson.

- Use the mid-point formula to calculate the centre of the line joining A and B.
- Join the point B to the point (4, 3).
- Find the mid-point of this line segment.



Practise your skills

- Practice sheet G7

Activity **Hills and Valleys** **Code G8**

This activity links to award learning outcome 2.4

Introduction

This activity will help you know how to work out how steep a hill is. Sometimes, when planning a car journey, it can be important to know how steep any hills are. You might have to avoid steep hills, especially if a trailer or caravan is attached to the car. The steepness of a hill is measured using the slope of a line from coordinate geometry.

Materials you will need
Pen and paper

Learning Outcomes

1. Find the slope of a line.

Key Learning Points

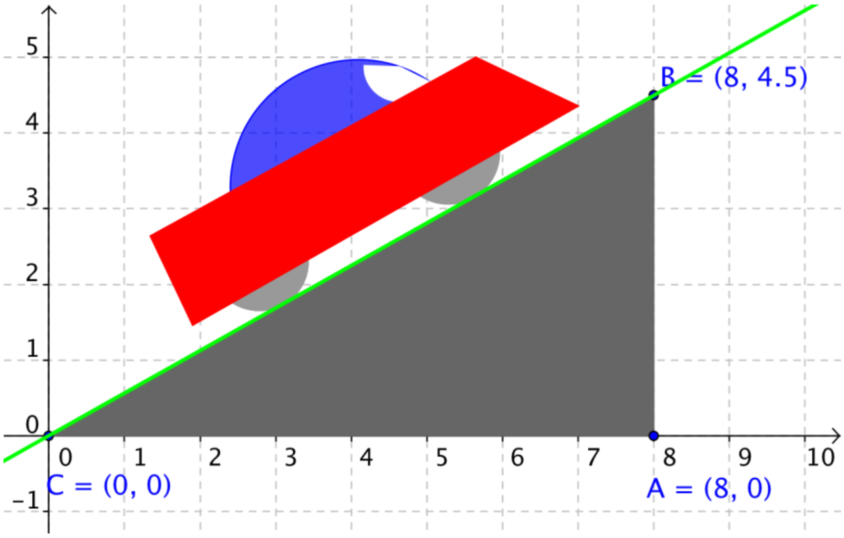
1. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

What do you need to know before you start?

You need to know:

how to read the coordinates of points in the plane;
how to measure horizontal and vertical distances;
how to read contour levels on a map;
how to get real distances from a map.

Task 1: Calculating the slope of a hill



In the drawing above the car and hill are superimposed on the coordinate plane. A line has been drawn to run along the hill. You can see that the height of the hill (AB) is 4.5 and the width into the hill to go up by this height (CA) is 8.

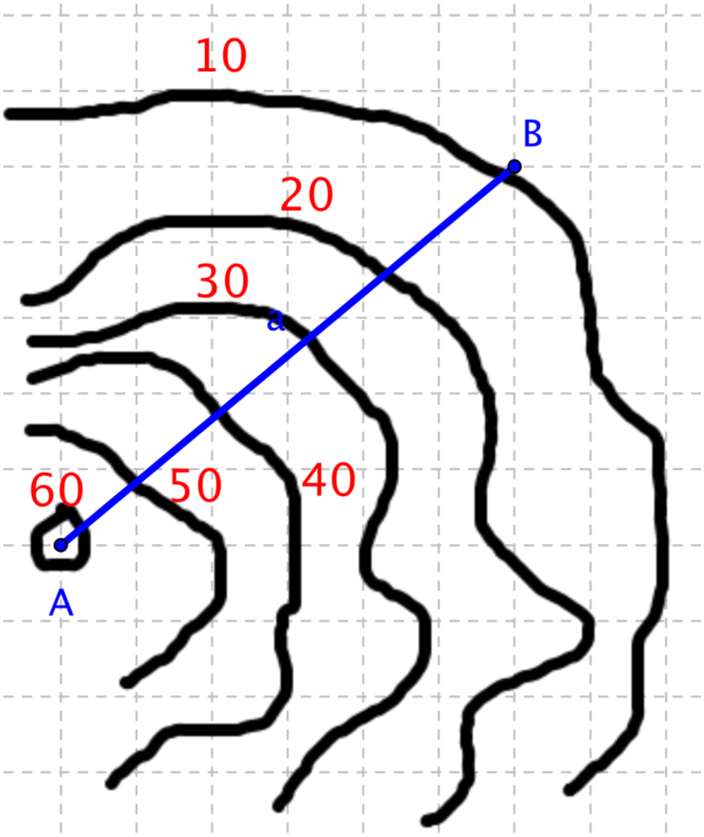
$$\begin{aligned} \text{slope} &= \frac{\text{height}}{\text{width}} \\ &= \frac{4.5}{8} \\ &= 0.5625 \\ &\approx 0.6 \end{aligned}$$

The slope formula will give you the same answer. You must use the two points C and B which are on the line.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4.5 - 0}{8 - 0} \\ &= \frac{4.5}{8} \\ &\approx 0.6 \end{aligned}$$

Now you try this

The diagram below shows a map with a road, BA, going up a gentle hill. The contour levels are marked on the map at 10 metre intervals. The scale of the map is 1cm = 100 m.



- What is the elevation of the top of the hill, A, above B?
- What is the horizontal distance from B to A?
- What is the slope of the road BA?

Practise your skills

- Practice sheet G8

Activity **Optical Illusions** **Code G9**

This activity links to award learning outcome 2.4.

Materials you will need

Pen and paper

Learning Outcomes

1. Tell when two lines are parallel.

Key Learning Points

1. If two lines are parallel their slopes are equal:

$$m_1 = m_2$$

What do you need to know before you start?

You need to know how to calculate the slope of a line segment.

You need to know that the slope of a line segment is given by the formula

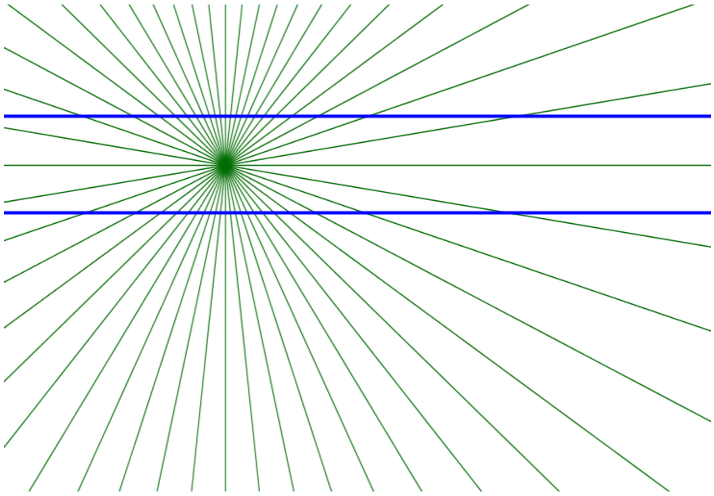
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Task 1: Optical illusions?

Example

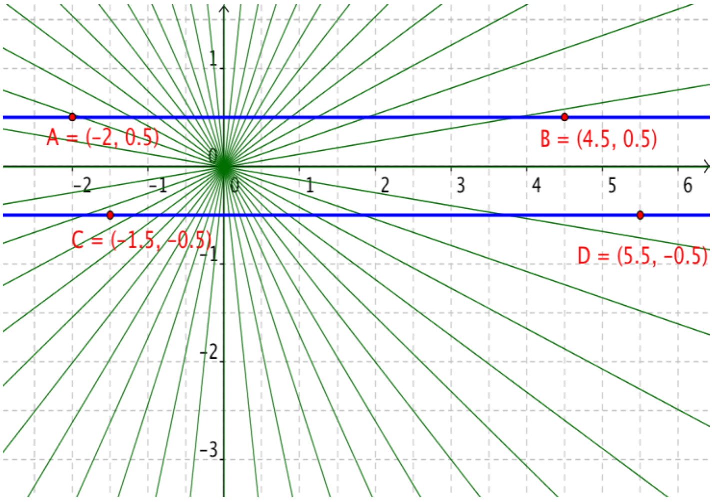
Optical illusions have entertained people for centuries. They still have the power to give us hours of pleasure.

Are the blue lines parallel?



Solution

Superimpose the diagram onto a coordinate plane. This allows us to get points on the two lines. When we have the points we can **get the slope of each line**. **Then we will know whether they are parallel or not.**



A is the point (-2, 0.5). B is the point (4.5, 0.5). The slope of the line AB is

$$\begin{aligned} m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0.5 - 0.5}{4.5 - (-2)} \\ &= \frac{0}{6.5} \\ &= 0 \end{aligned}$$

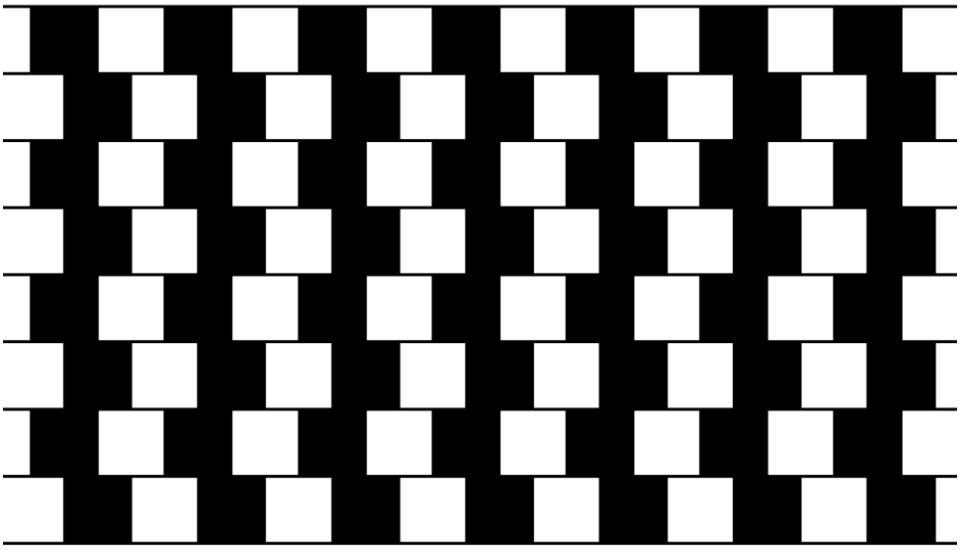
C is (-1.5, -0.5) and D is (5.5, -0.5) so the slope of CD is

$$\begin{aligned} m_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-0.5 - (-0.5)}{5.5 - (-1.5)} \\ &= \frac{0}{7} \\ &= 0 \end{aligned}$$

The slopes are equal so the lines are parallel despite the illusion.

Now you try this

The diagram below shows a grid that is difficult to focus on. Are the horizontal lines parallel or not?



Solution

- Pick a point in the grid as origin and draw a coordinate plane over it.
- Find the coordinates of two points on each horizontal line.
- Use the slope formula to calculate the slopes of the lines.
- Hence show that the lines are parallel.

Practise your skills

- Practice sheet G9

Activity **Playing Pool** **Code G10**

This activity links to award learning outcome 2.4 and 2.5

Introduction

Pool is a very popular game in Ireland. Many people play it and many more are spectators and watch it on television. Many shots rely on a player being able to draw perpendicular lines mentally. Without this ability many shots would be either impossible or very difficult.

Learning Outcomes

1. Tell when two lines are perpendicular.

Key Learning Points

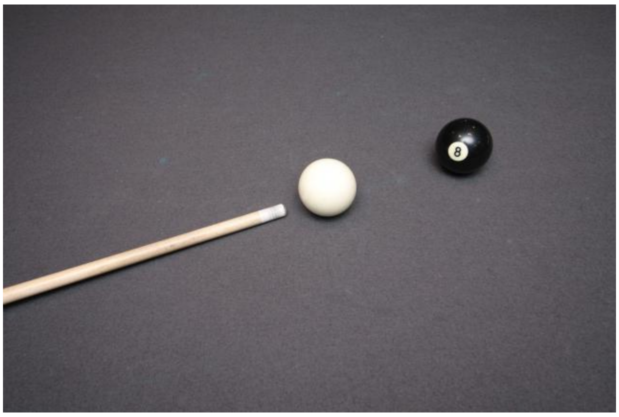
1. If two lines are perpendicular the product of their slopes is –1:

$$m_1 \times m_2 = -1$$

What do you need to know?

You need to know how to calculate the slope of a line segment given two points on it.

You need to know how a ball is struck in pool or billiards. Watch a game in action or, if you are a player, explain the game to others.

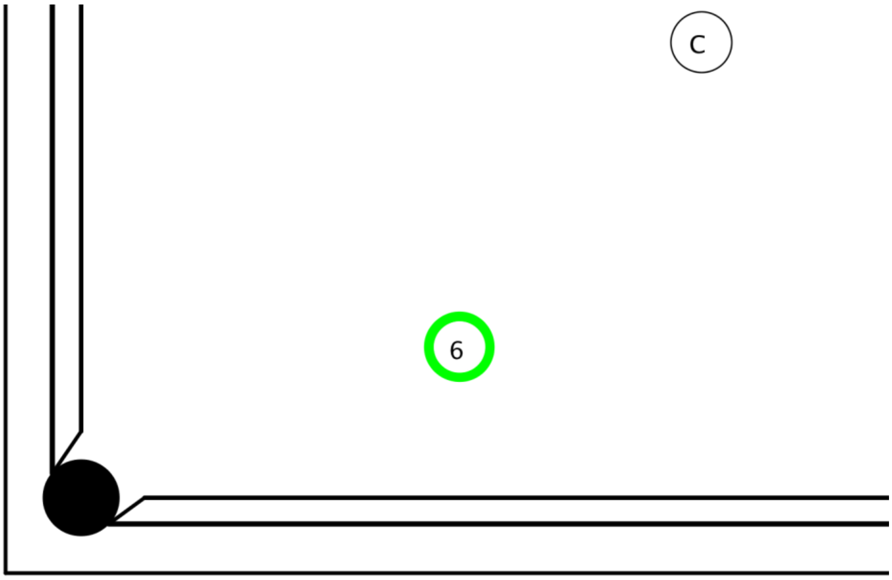


(Image:Suvro Datta /FreeDigitalPhotos.net)

Task 1: Potting the ball

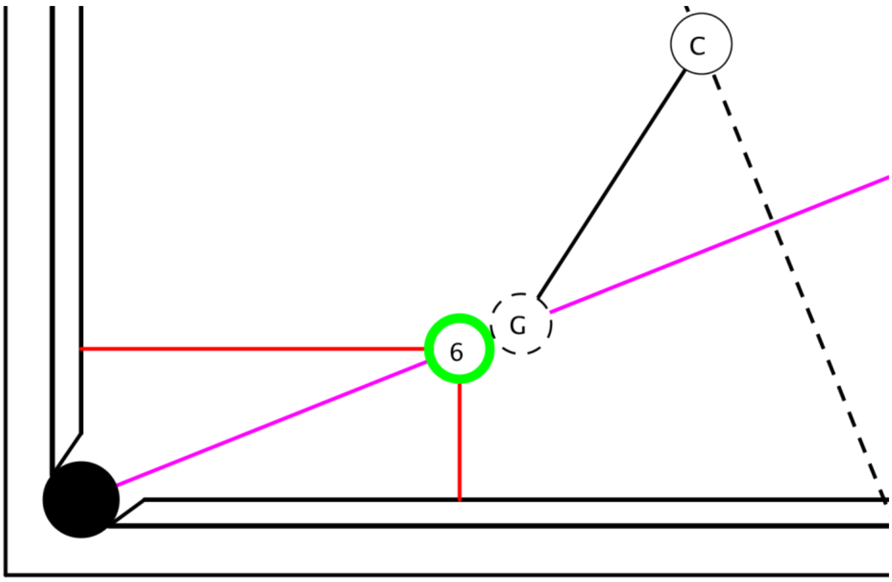
Example

In the diagram below how should a player strike the cue ball, C, in order to sink ball number 6 into the pocket at the lower left?



Solution

Mentally, the player should construct a rectangle. The player should imagine two straight lines going from the centre of ball 6 perpendicular to the sides of the table. Draw the diagonal of this rectangle and imagine a ‘ghost’ ball, G, touching ball 6 on this diagonal. The player should then aim the cue to strike ball C in order that it will hit the ghost ball.



If the slope of the diagonal is $\frac{2}{5}$ what is the slope of the dashed line which is perpendicular to it?

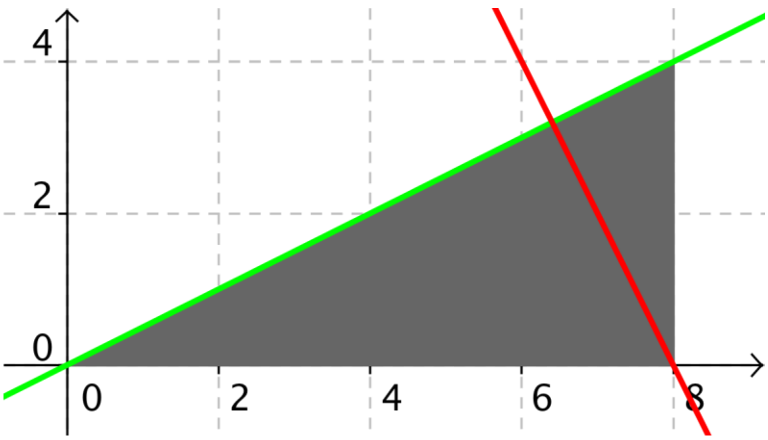
Solution: $m \times \frac{2}{5} = -1 \Rightarrow m = -1 \times \frac{5}{2} = -\frac{5}{2}$

Now you try this

- Look at the diagram of the pool game on the previous page. If the slope of the diagonal were $\frac{2}{3}$ what would be the slope of the dashed line?

- If the dashed line had a slope of $-\frac{3}{7}$ what would be the slope of the diagonal?

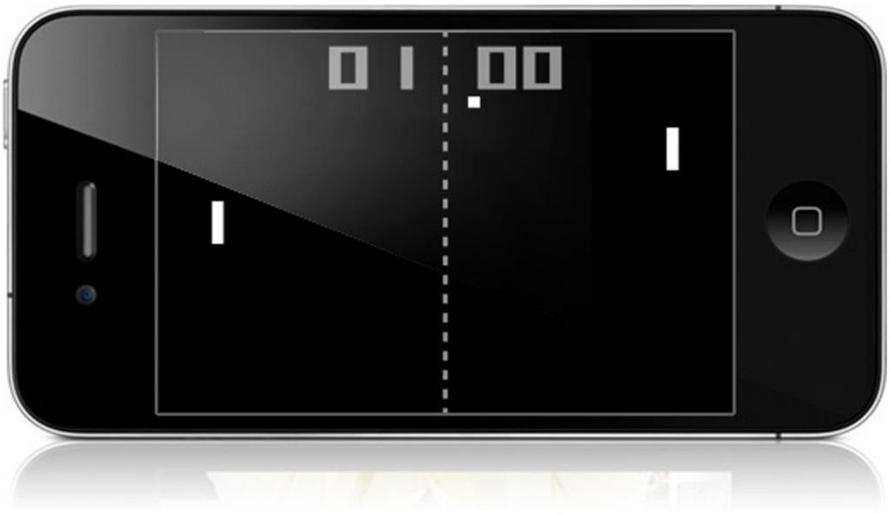
- A car is going up a steep hill, represented by the green line in this diagram. The green line has a slope 0.5. **What is the slope of the red line** drawn perpendicular to the hill's surface?



Practise your skills

- Practice sheet G10

Activity **Playing Pong on the iPod** **Code G11**



This activity links to award learning outcome 2.4.

Introduction

Pong is one of the original Arcade games. A copy is normally included with the iPod. If you have not played it before or you do not have it on an iPod you can play it if you have java enabled on your computer, at http://www.xnet.se/javaTest/jPong/jPong.html

Learning Outcomes

1. Write the equation of a straight line.
2. Interpret the equation of a straight line.

Key Learning Points

1. The equation of a straight line with slope *m* passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

What do you need to know?

The slope, *m*, of a line is the vertical distance divided by the horizontal distance between any two points on the line. This is fundamental to every computer game because it is the method used to calculate movement on a screen.

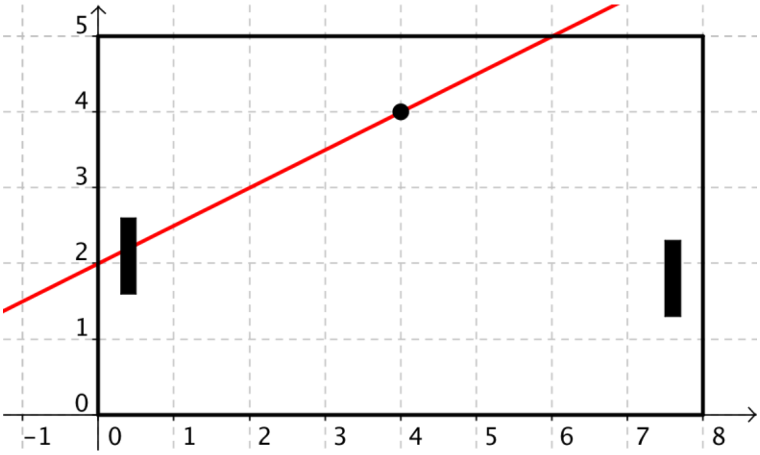
The ball is controlled by the two bats which strike the ball and send it moving in a straight line. The object of the game is to prevent the ball from striking either end wall.

Task 1: Playing Pong

Example

Look at the diagram below. This version of Pong is played on a rectangular grid which is 8cm wide by 5cm high. To look at the motion of the ball we will set the origin at the bottom left corner. After one encounter with the bat on the left, the ball is moving along a line which passes through the two points (0,2) and (6, 5).

What is the equation of this line?



Solution

First we must get the slope of the line.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-2}{6-0} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

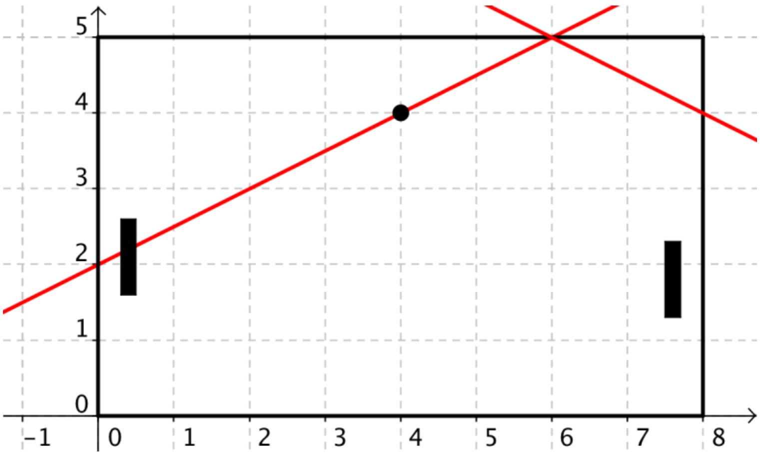
We can use this value of the slope and the coordinates of either point to get the equation of the line.

$$\begin{aligned} & y - y_1 = m(x - x_1) \\ & \Rightarrow y - 2 = \frac{1}{2}(x - 0) \\ & \Rightarrow 2y - 4 = x \\ & \Rightarrow 2y = x + 4 \end{aligned}$$

Now you try this

- After the ball bounces off the wall it **follows the straight line connecting the points (6, 5) and (8, 4)** as in the diagram.

  What is the equation of this line?



- At a later stage in the game the ball bounces off the right wall. It follows a line whose equation is $3x - 4y = 12$.

  Where does the ball meet the bottom wall?

Practise your skills

- Practice sheet G11

Activity — Designing a Flower Bed — Code G12

This activity links to award learning outcome 2.4.

Introduction

People have always loved gardens. Flowers, in particular, appeal to people all over the world. One of the most common arrangements of flowers in a garden is to put them in circular beds.

Learning Outcomes

1. Write the equation of a circle.
2. Interpret the equation of a circle.

Key Learning Points

1. The equation of a circle centred at the origin with radius *r* is given by the equation $x^2 + y^2 = r^2$

What do you need to know?

Maths

The coordinates of the origin are (0, 0). A circle is a shape traced out by a point that is always a fixed distance from a given point. This distance is called the radius. The distance between two points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

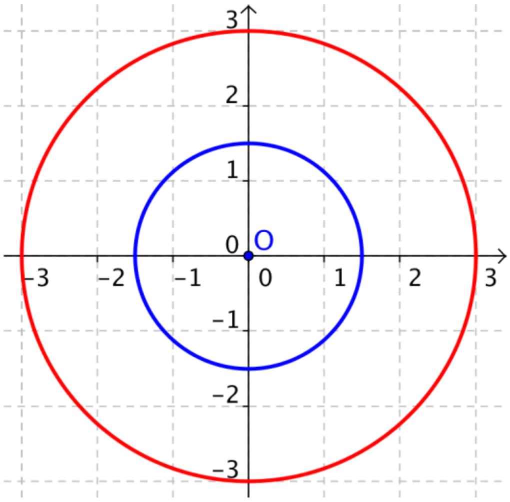
Flower beds

In planning a display the first thing to do is draw out the design. You think of the colours you want to mix. You think of the order in which you want to see them. You plan the size of the circles that they will occupy.

Task 1: A circular flower bed

Example

1. In designing a flower bed with two circles of flowers we want the outer red circle to have a **radius** of 3. **What is the equation of the outer circle?**

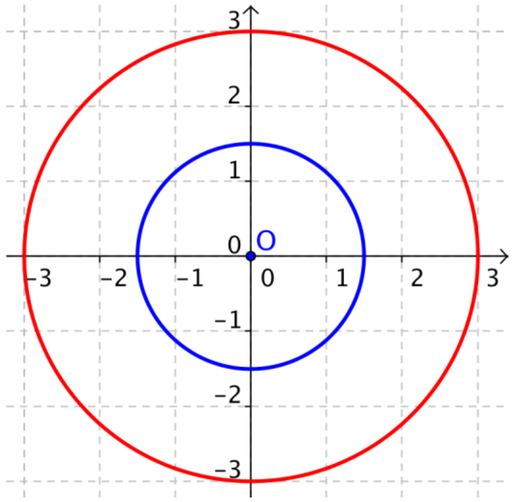


Solution

We know that the radius is 3. So the equation is

$$x^2 + y^2 = r^2$$
$$\Rightarrow x^2 + y^2 = 3^2$$
$$\Rightarrow x^2 + y^2 = 9$$

2. The inner blue circle has the equation $4x^2 + 4y^2 = 9$. **What is the radius of the inner circle?**



Solution

First we must write the circle in the form $x^2 + y^2 = r^2$ because the number at the right hand side of the equation will then be the square of the radius.

$$\begin{aligned} &4x^2 + 4y^2 = 9 \\ &\Rightarrow x^2 + y^2 = \frac{9}{4} \\ &\Rightarrow x^2 + y^2 = \left(\frac{3}{2}\right)^2 \\ &\Rightarrow r = \frac{3}{2} \end{aligned}$$

Now you try this

- We want to add **another circle** of flowers of **radius 2**. What is the equation of the circle they will occupy?
- Another layer of flowers is planted to extend the size of the flower bed. The equation of the circle is $16x^2 + 16y^2 = 169$. How big is this circle?

Practise your skills

- Practice sheet G12

Activity **Playing Shove Ha'penny** **Code G13**

This activity links to award learning outcome 2.4.

Introduction

The game of Shove Ha'penny is very old and has been played with many different coins throughout the world.

Learning Outcomes

1. Get the equation of a tangent to a circle.
2. Discover where a tangent meets a circle.

Key Learning Points

1. Equation of a tangent to a circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$
2. A tangent $xx_1 + yy_1 = r^2$ meets the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1)

What do you need to know?

The equation of a circle centered at the origin is $x^2 + y^2 = r^2$.

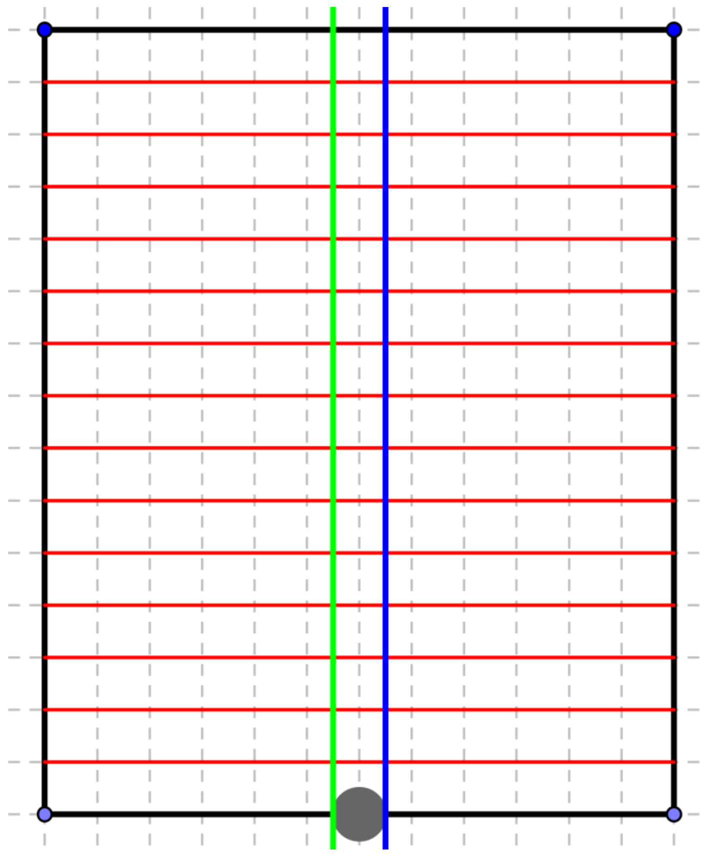
You need to know the rules of Shove Ha'penny. You will play the game with cents instead of ha'pennies.

Task 1: The shove ha'penny board

Example

You will need to make your own board to ensure that it can be used with cents which are not the same size as ha'pennies. The vertical lines you put on the board will be **tangents** to the circles represented by the cents.

Look at the diagram below. A cent is placed on the board and we coordinate the board so that the **origin** is the centre of the coin and the diameter of the cent is 1 unit. The red horizontal lines are the lines between which the coin must finish.



The green and blue lines going up the board must be tangents to the circle.

1. What is the equation of the circle?
2. Where does the green line meet the circle?
3. What is the equation of the green line?

Solution

1. We know that the diameter of circle is 1 so its radius is $\frac{1}{2}$. The equation is

$$x^2+y^2=r^2$$
$$\Rightarrow x^2+y^2=\left(\frac{1}{2}\right)^2$$
$$\Rightarrow x^2+y^2=\frac{1}{4}$$
$$\Rightarrow 4x^2+4y^2=1$$

2. If the radius of the circle is $\frac{1}{2}$ the green line meets the circle at $\left(-\frac{1}{2},0\right)$.
3. The equation of the green line, which is a tangent to the circle, is

$$xx_1+yy_1=r^2$$
$$x\left(-\frac{1}{2}\right)+y(0)=\left(\frac{1}{2}\right)^2$$
$$-\frac{x}{2}+0=\frac{1}{4}$$
$$x=\frac{-2}{4}$$
$$x=-\frac{1}{2}$$
$$2x+1=0$$

Now you try this

- Where does the blue line meet the circle?
- What is the equation of the blue line?

Practise your skills

- Practice sheet G13

Activity **Creating a Pattern** **Code G14**

This activity links to award learning outcome 2.5.

Introduction

Tessellations in the Alhambra http://www.flickr.com/photos/gruban/11341048/

The picture above is some of the art drawn in the Alhambra in Spain. It is heavily reliant on geometric patterns. Much of the art that is around you in everyday life– such as wallpaper, mosaics or other decorations – is also based on geometry.

Materials you will need

Pen and paper
Mathematical instruments
Coloured markers
Access to internet

Learning Outcomes

1. Construct geometric shapes.

Key Learning Points

1. Simple constructions

What do you need to know?

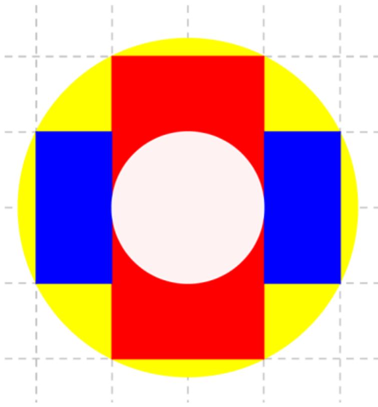
You need to be able to use mathematical instruments*:* compass, ruler, protractor and set squares.

You need to practise breaking down a geometric figure into simpler parts.

The distance between two points is $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Task 1: Designs

Example



This is a shape that is part of a larger design for wallpaper in a child's room. The shape can be rotated, scaled up and down and repeated. Once we are able to draw this shape then the wallpaper can be designed.

The shape consists of two circles and two rectangles. The grid lines show us the correct proportions.

If the radius of the inner circle is one unit:

1. What are the dimensions of the red rectangle?

2. What are the dimensions of the blue rectangle?

3. What is the radius of the outer circle?

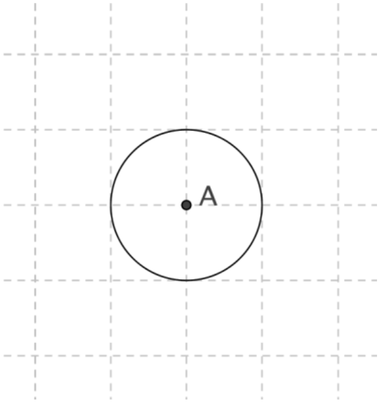
4. Construct the shape.

Solution

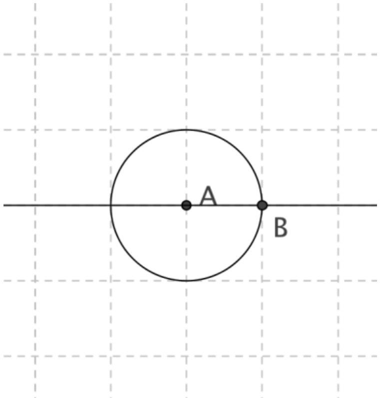
1. The red rectangle is two units wide by four units high.
2. The blue rectangle is four units wide by two units high.
3. The outer circle joins the point $(0,0)$ to $(1,2)$ so the radius is

$$r = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5}$$

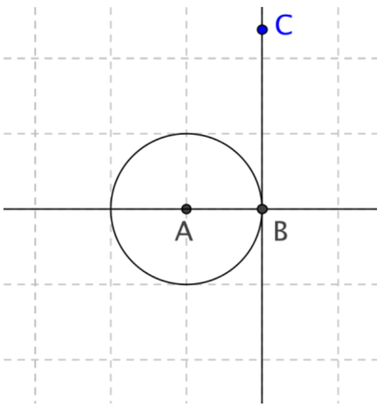
4. Get your compass. Use a ruler to set the radius to draw a circle:



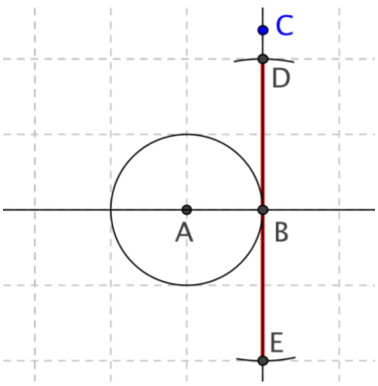
Use your ruler to draw a horizontal line AB through A, the centre of the circle, to cut the circle at the point B.



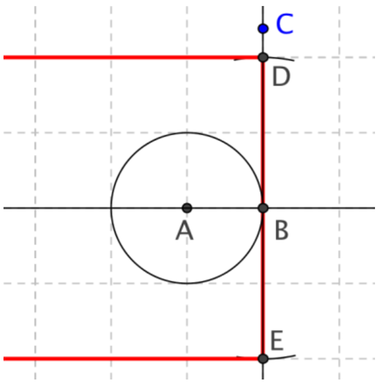
Use your set square to draw the line BC at the point B which is perpendicular to the line AB.



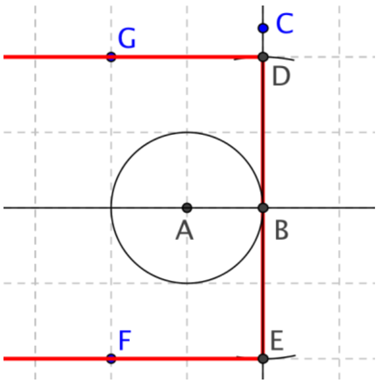
Use your ruler to adjust the radius of the compass to 2. With the centre of the compass at the point B mark the points, D and E, two units above and below the point B.



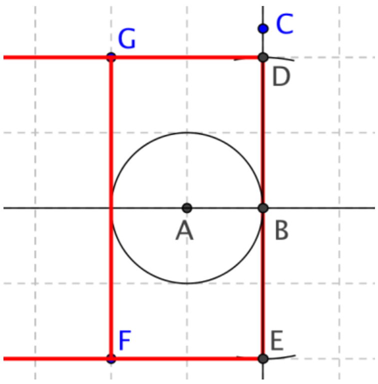
With your set squares draw perpendiculars to BC at D and E.



Now use your compass again to draw the points G and F two units to the left of D and E.



Join G and F.



Now you try this

- Use similar methods to construct the blue rectangle.
- Note that the corners of both rectangles are on the circumference of the outer yellow circle. Use this information to construct the outer circle.
- Colour in the pattern with colours of your choice.

Practise your skills

- Practice sheet G14

Activity **Measuring a Room** **Code G15**

This activity links to award learning outcome 2.6.

Introduction

When painting or putting up wallpaper you need to calculate how much you will need. Before you start the job it is important to be able to calculate whether you will have enough material to finish it. It might be even more important to calculate whether you have enough money to purchase the raw materials. Since paint covers the area of a wall you need to find the area to estimate how much paint, and hence the cost, will be required.

Learning Outcomes

1. Calculate the area of a rectangle.
2. Calculate the length of the perimeter of a rectangle.

Key Learning Points

1. The formula for area of a rectangle is $A = l \times b$.
2. The formula for perimeter of a rectangle is $P = 2(l + b)$.

What do you need to know?

The formula for the area of a rectangle of length l and breadth *b* is $A = l \times b$**.**

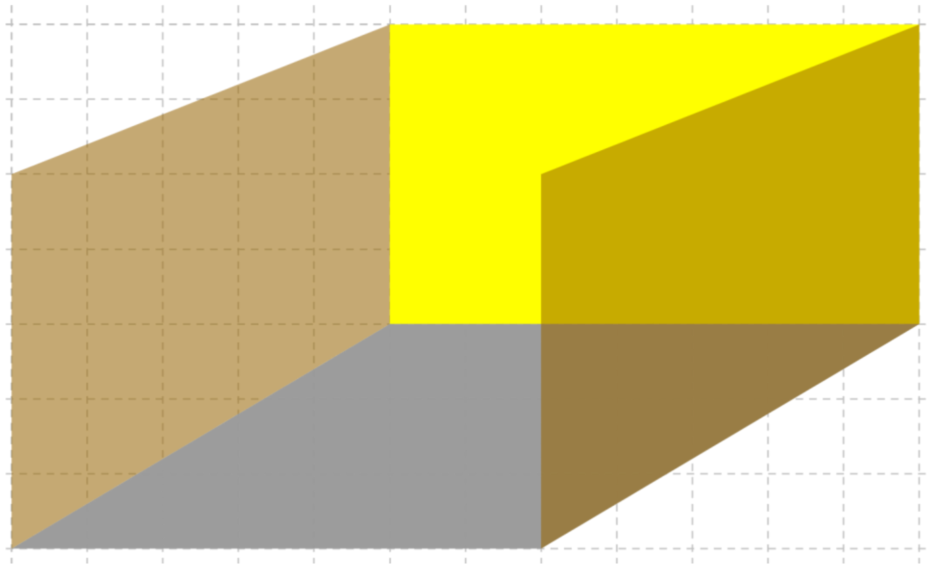
The formula for the perimeter of a rectangle of length l and breadth *b* is $P = 2(l + b)$.

You need to be able to **use a measuring tape** to get the length of a wall.

You need to be able to **calculate area** from length and breadth.

Task 1: Painting a room

Example



The drawing above shows a room which needs painting. The yellow wall measures 3.6 metres long by 2.7 metres high. You want to paint this blue. You can buy the paint in 2.5 litre containers. One litre of paint will cover $8.6m^2$ of wall. Each container costs €16.87 in a sale. You need to put masking tape on the wall to protect the other walls, floor and ceiling.

How many containers of paint should you buy if the wall will need two coats?

How much will the paint cost?

How much masking tape do you need?

Solution

1. One container of paint would be enough.

Calculate the area of the wall as follows:

$$\begin{aligned} A &= l \times b \\ &= 3.6 \times 2.7 \\ &= 9.72\,m^2 \end{aligned}$$

Since you need two coats of paint you must double this value:

Total Area = $19.44m^2$

Convert this **area** measure **to the volume** of paint needed:

$$\begin{aligned} 8.6m^2 &= 1l \\ 1m^2 &= \frac{1}{8.6}l \\ 19.44m^2 &= \frac{19.44}{8.6}l \\ 19.44m^2 &= 2.26l \end{aligned}$$

So, one container will be enough.

2. The cost is €16.87.

3. The perimeter of the wall is $2(l+b)$, so the length of masking tape required is:

$$2(l+b) = 2(3.6+2.7) = 2 \times 6.3 = 12.6m$$

Now you try this

- The side walls are 4.3 metres long. Calculate the area of a side wall.
- Each wall will need two coats of paint. How much paint is needed for the side walls?
- What is the cost of painting the side walls?
- How much masking tape is needed for the side walls?

Practise your skills

- Practice sheet G15

Activity **Buying a Ladder** Code G16

This activity links to award learning outcome 2.8.

Introduction

This activity is an example of how we can use geometry to solve real-life problems.

Learning Outcomes

1. Use similar triangles to calculate the height of a house.
2. Use the Theorem of Pythagoras to calculate the length of the required ladder.

Key Learning Points

1. The application of your knowledge of geometry to practical problems.

What do you need to know?

You need to know the axioms and theorems of geometry.

You need to know that if two triangles are similar the corresponding sides are in the same ratio.

You need to know that in a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

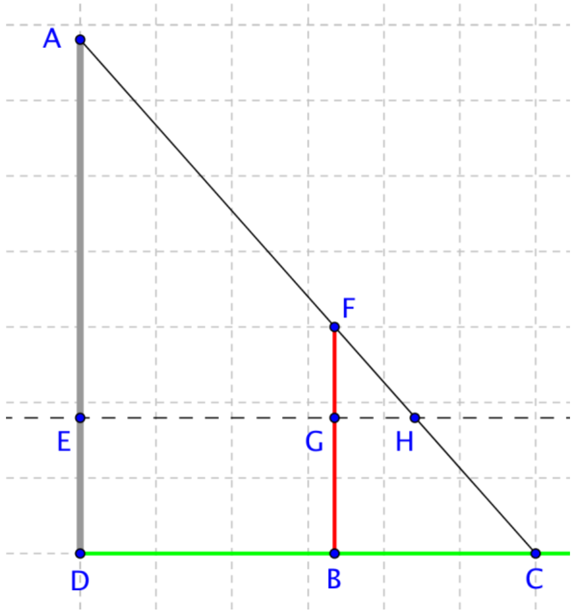
Task 1: Calculating length of ladder

Example

Picture by Gavin Brett, 2 November 2008, Moonah, Tasmania

The picture above shows a broken gutter on a house. You want to fix it yourself but you do not have a ladder long enough. You do not want to waste money on a ladder longer than you will need. **How can you calculate the length of the required ladder?**

Solution



1. A friend holds a post 3 metres high. You move back until as you look up the post just hides the gutter from your view. You have measured that your eyes are 1.8 metres above the ground. Your friend measures that the perpendicular distance from your eyes to the post, |HG|, is 1.06 metres. He also measures the perpendicular distance from G to the wall, |GE|, to be 3.35 metres. Since your eyes are 1.8 metres above the ground and the post is 3 metres high then the distance |GF| is 3 – 1.8 = 1.2 metres.

2.The triangle AEH and the triangle FGH are similar so

$$\frac{|AE|}{|FG|}=\frac{|EH|}{|GH|}$$

$$\frac{|AE|}{1.2}=\frac{4.41}{1.06}$$

$$|AE|=\frac{1.2\times 4.41}{1.06}$$

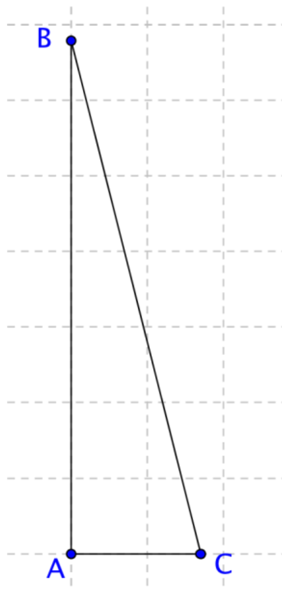
$$|AE|=4.99$$

$$|AD|=6.79$$

which is the height of the wall holding the gutter.

Health and safety considerations advise that a ladder should make an angle of 75° with the ground. This is often known as the 1 in 4 rule. The base of the ladder should be 1 unit out from the wall for every 4 units up. Since the wall is 6.79 metres high the base of the ladder should be 1.7 metres out.

How long should the ladder be?

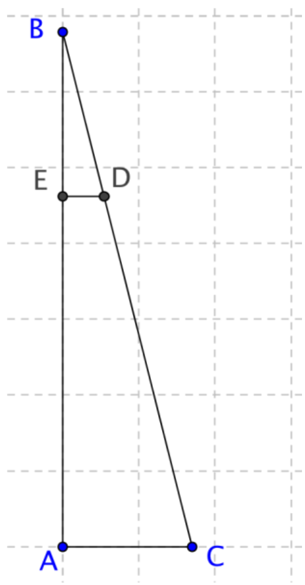


By the Theorem of Pythagoras we have

$$|BC|^2 = |AB|^2 + |AC|^2$$
$$|BC|^2 = 6.79^2 + 1.7^2$$
$$|BC|^2 = 46.1 + 2.89$$
$$|BC|^2 = 48.99$$
$$|BC| = \sqrt{48.99}$$
$$|BC| = 7$$

which means **the ladder has to be 7 metres long.**

Now you try this



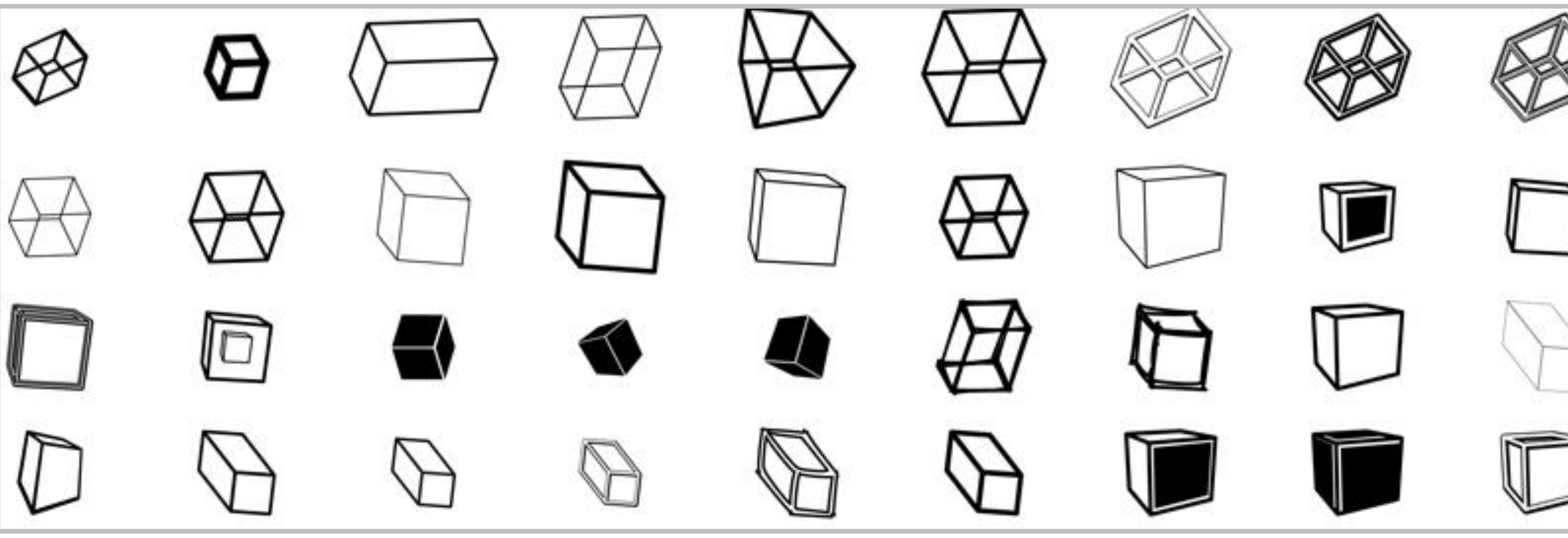
The ladder cannot be left leaning on the gutter as this is very unsafe. Instead a stand-off device is attached to the second rung, D, of the ladder from the top which is 0.56 metres down the ladder.

- Use similar triangles to calculate how the width, ED, of the stand-off device.
- Use the Theorem of Pythagoras to calculate BE, the distance below the gutter where the stand-off device meets the wall.

Practise your skills

- Practice sheet G16

Activity 3-D Shapes Code G17



This activity links to award learning outcome 2.7.

Learning Outcomes

- Solve practical problems by using the correct formulae to calculate the volume/capacity and surface area of a **cube, cuboid, cylinder, cone, and sphere**, giving the answer in the correct form and using the correct terminology.

(FETAC Award Specification Mathematics Level 4, Learning Outcome 7)

Your tutor will provide you with materials and will guide you through a series of activities and tasks to help you learn about cubes, cuboids, cylinders, cones and spheres.
You will also find useful material on websites such as
http://www.onlinemathlearning.com/geometry-help.html.

Acknowledgements

This Learner Pack and the accompanying Tutor Guide were commissioned by FAS to assist learners in FAS Community Training Centres (CTCs) to develop knowledge, skills and competence in mathematics and to achieve FETAC certification in Level 4 Mathematics.

Similar resources were developed to support CTC learners in working towards FETAC certification in Level 3 Application of Number.

The **National Adult Literacy Agency** (NALA) coordinated the resources development and the associated staff training programme, and edited the packs. The resources were developed by a team from NALA and the **National Centre for Excellence in Mathematics and Science Teaching and Learning (**NCE-MSTL), and the project was initiated, funded and managed by FAS.

We would like to thank the learners, staff and manager of Newbridge Youth Training and Development Centre for trying out the level 3 materials in the early stages of development.

Thanks also to CTC staff who participated in the training days in Real World Maths in January 2011, and who gave valuable feedback on the draft resources for Level 3 and Level 4.

FAS:

John O'Neill
Louise McAvin
Fionnuala Anderson

NALA:

Bláthnaid Ní Chinnéide
Fergus Dolan
John Stewart
Dr Terry Maguire (Institute of Technology Tallaght)

NCE-MSTL:

Prof. John O'Donoghue
Dr. Miriam Liston
Dr. Niamh O'Meara
Tim Brophy
Dr. Mark Prendergast
Paraic Treacy
Dr. Lisa O'Keeffe

2011

NDP
National Development Plan
Transforming Ireland

FÁS
Foras Áiseanna Saothair
Training & Employment Authority

Ireland's EU Structural Funds Programmes 2007 - 2013

Co-funded by the Irish Government and the European Union

EUROPEAN SOCIAL FUND